

This exam has FOUR (4) Questions for a total of 100 Points

⇒ Please answer **ALL** questions + **ALL Answers MUST Be Justified** ⇐
(answers alone are **not sufficient**)

⇒ **PLEASE RECORD ALL ANSWERS IN THE ANSWER BOOKLET** ⇐
(Any material not in Answer Booklet(s) **will not be seen.**
In particular, the exam paper **will not be graded** or reviewed.)

For reference, the exam consists of questions based on robots topics such as:

1. Position, orientation and location in space
2. Robot analysis
(forward/Inverse kinematics, recursive Newton-Euler formulations, etc.)
3. Sensing geometry (including camera calibration)
4. Multiple-view geometry
5. (Deterministic) Motion planning and control

1. **Transforming 3D Space** (15 Points)

For a general kinematic frame in 3D space, the homogeneous transformation matrix is a 4×4 matrix that naturally partitions into four (4) submatrices as:

$${}^A T_B = \begin{bmatrix} [A] & [B] \\ [C] & [D] \end{bmatrix}$$

- A. For each submatrix please specify specifically what the components (i.e., [A], [B], [C], [D]) functionally are including their dimensions.
- B. For the case of Euclidian, Rigid-Body Transformations, what simplifications and constraints can be said for these components (i.e., [A], [B], [C], [D]) ?
- C. Is it ever the case that a Transformation Matrix is orthogonal?
That is:

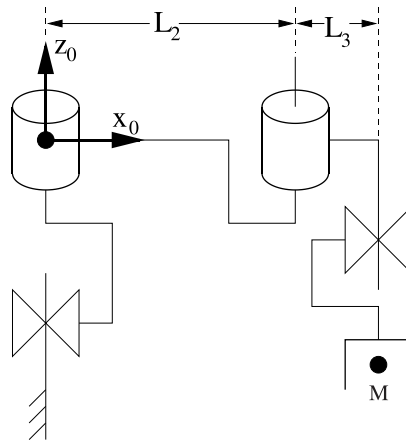
$$({}^A T_B)^{-1} \stackrel{?}{\Leftrightarrow} ({}^A T_B)^T$$

If so, what constraints/assumptions are needed with respect to the components (i.e., [A], [B], [C], [D])?

If not, why?

2. **A New SCARA Need Not Scare** (50 Points)

A PRRP manipulator with four parallel joint axes is proposed as an alternative to the traditional SCARA (Selective Compliance Assembly Robot Arm) RRRP robot for pick and place of a point mass object (noted as M). Links 1 and 4 (L_1 and L_4) vary in length as they are prismatic; links 2 and 3 are constant. A simple schematic is below:



A. Sketch the workspace of this manipulator. Please assume $L_2 > L_3$ and no rotary joint limits. **Be sure to include dimensions in your drawing.**

B. What are the D-H Parameters for this robot?

Joint i	θ_i	d_i	a_i	α_i
1				
2				
3				
4				

C. Please determine the Forward Kinematics for this robot.

D. With no joint limits, if we are considering only the position of the end effector, how many inverse kinematic solutions are there (in general)?
 (Hint: the inverse kinematics does not need to be solved explicitly to answer this).

E. Can this manipulator archive arbitrary orientations in its workspace (i.e., is it dextrous)?

F. Does the arm have redundancy? If so, how many DOFs are redundant?

G. Please derive the Jacobian, J , for this robot.

H. Briefly discuss how an “Explicit form” of the dynamics (i.e., also known as a Recursive Newtown Euler Approach) can be used to recover the equations of motion for the arm.

(Please assume: No external forces/moments, negligible joint friction, and the links are homogenous with small cross-sections and thus have inertias given by $\frac{1}{12}ml^2$).

I. Please provide a general means to determine the external forces, F , for a set of motor forces/torques given as a vector, τ .

(Hint: The answer may be In general terms of the Jacobian, J).

J. As the mass (M) might be heavy, a gear box is proposed for the joint.

Briefly discuss the influence of this gear box on the robot dynamics.

(Hint: What is the effect of the gear ratios on the reflection of the inertia in the joint space?)

3. **Taking Perspective of Perspective Geometry** (15 Points)

Consider the projective transformation, H , of an image, I , with four (4) equidistant collinear image features a, b, c, d (with corresponding points in 3D space A, B, C, D).

- A. Is it true that under a perspective projection of the Image I to I' , that the projected features in I' (i.e., a', b', c', d') are also collinear in this new 2D image plane? **Please briefly state why.**
(Hint: Is collinearity invariant under H ?)
(Note: an extended proof/disproof might be challenging and is not required)
- B. Will the observed features (i.e., a', b', c', d' from Part (A)) also be equidistant in the new image plane I' ? **Please briefly state why.**
- C. Does this extend to Structure from Motion (SfM) and/or stereography? Specifically, in a subsequent/second image I^* will a set of corresponding features a^*, b^*, c^*, d^* (cast from the corresponding 3D points A, B, C, D) **be collinear** taken as the camera is moved (or, for the stereo case, from the “other” camera view)? **Please briefly state why.**

4. **Truth in Robotics** (20 Points)

Please indicate, and **very briefly state why**, the following statements are either:

- Surely **TRUE**
 - Surely **FALSE**
 - **INDETERMINATE** (i.e. cannot be determined)
- A. All rotations of a frame in three-dimensional Euclidean space \mathbb{R}^3 may be represented in the group $SO(3)$.
- B. Any valid affine transformation is also a valid perspective transformation.
- C. The fundamental matrix is invertible.
- D. A manipulator’s configuration space is greater or equal to its workspace.
($\dim(C_{space}) \geq \dim(W)$)
- E. Motion planning for a closed kinematic chain, such as that which occurs if multiple robots contact and manipulate the same body, is **easier** to solve than if the same number of chains were open.

END OF EXAMINATION — Thank you ☺