⇒ Sort Pattern: **A** ⇐

METR 4202 -- Advanced Controls & Robotics

Individual Quiz September 23,2015

Name:_____

Student Number:_____

This quiz consists of **Multiple Choice**, **Short Answer**, and **Worked Problems.** Please Answer **All** Questions below on the quiz paper. Answers **must be neat and clear. All answers (except for multiple choice) must provide a brief** justification.

You may use the back of each sheet as scratch paper if needed. Each question is worth **10 points. The total quiz is worth 100 points.**

Please clearly mark your final answer

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1. Consider a general 2D transformation (e.g., for images) where **x'** is a point in the second image and **x** is a point in the first image.

   a. If we assume the transformation is **rigid (Euclidean)**, how many point correspondences (i.e., **x'** and **x** together) are needed to recover the transformation?

   b. If we assume the transformation is a **similarity transformation**, how many point correspondences (i.e., **x'** and **x** together) are needed to recover the transformation?

   c. If we assume the transformation is an **affine transformation**, how many point correspondences (i.e., **x'** and **x** together) are needed to recover the transformation?

2. What are the differences between the joint space, workspace and operation space? (explain fundamentally, not just their definitions).

3. What does the Null-space of the Jacobian inform or imply for a serial robot?

4. What is a distinguishing feature of SfM's operation, in particular over SLAM? (explain fundamentally, not just the acronyms).

5. Consider two 2D homogenous points $\mathbf{P_1}$ and $\mathbf{P_2}$. For parts (a), (b) and (c), what is the equation of the line between them?

   a. Two non-collinear points If $\mathbf{P_1}$=**(a, b, c)** and $\mathbf{P_2}$=**(d, e, f)**

   b. If $\mathbf{P_1}$=**(3, 2, 1)** and $\mathbf{P_2}$=**(2.5, 2, 1)**

   c. If $\mathbf{P_1}$=**(4, 2, 1)** and $\mathbf{P_2}$=**(0, 2, 1)**

   d. Are the lines found in part (b) and (c) parallel to each other? Briefly, why?

6. Consider the camera calibration process with a *planar* calibration object (e.g., a planar checkerboard). The object possesses **M** distinct features. The camera takes **K** images.

   You may assume an undistorted camera with following following characteristics:
   - Zero skew (orthogonal pixel arrangement)
   - Unity aspect ratio (square pixels)
   - Known image center (at ½ of image height and width)

   a. Given these assumptions, what unknown parameters are there to calibrate?
   (hint: what intrinsic parameters need to be recovered? And, what extrinsic parameters need to be recovered for each image?)

   b. For the K images, each of M features, how many constraints are given?

   c. For the K images, each of M features, how many parameters need to be calibrated?

   d. For M = 4, what is the minimum number of images needed to determine the calibration?

7. Please state if the following statements are generally **TRUE (T)** or **FALSE (F)**

   a. The inverse of a rotation matrix is always its transpose. **T | F**

   b. The inverse of a transformation matrix is its transpose. **T | F**

   c. Homogeneous transforms are linear operations. **T | F**

   d. In DH one of the four parameters (a, α, d, θ) must be 0. **T | F**

   e. The inverse kinematics of a 6R arm is closed form with 16 solutions. **T | F**

   f. Straight lines remain straight under a perspective transformation. **T | F**

   g. For a manipulator, the torque needed is a function of the pose. **T | F**

   h. RGB colour spaces are invariant to changes in illumination. **T | F**

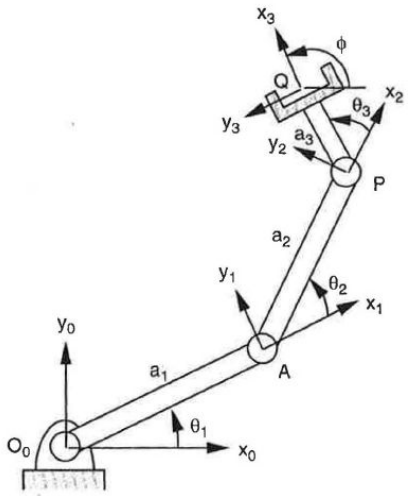
   i. Local perspective transformations are approximately affine transformations. **T | F**

   j. The fundamental matrix is invertible. **T | F**

8. What small changes in properties is SIFT invariant to?
   a. Rotation
   b. Illumination
   c. Affine
   d. (a) and (b)
   e. (a) and (b) and (c)

9. For a general coordinate transformation between four frames {A}, {B}, {C}, and {D}. What is the overall transformation matrix ${}^{A}_{D}T$ between {A} and {D} as a function of the **individual** Rotations [${}^{i+1}_{i}R$, eg ${}^{a}_{b}R$] and Positions [${}^{i+1}_{i}P$, eg ${}^{a}_{b}P$].

10. Consider the planar manipulator shown at right



Assume the following properties:

- $\mathbf{a_1}$ = 42 cm
- $\mathbf{a_2}$ = 32 cm
- $\mathbf{a_3}$ = 15 cm
- $\boldsymbol{\theta_1}$, $\boldsymbol{\theta_2}$ and $\boldsymbol{\theta_3}$ may take a value from 0 to 360°

a. Provide two classes of pose where the arm will be at a singularity

b. What value of joint angles will get the tip to the position [0, 89] cm?

c. What value of joint angles will get the tip to the position [21.4, 53] cm with an orientation of 180 degrees (in Frame {0})?