



Robot Sensing: Feature Detection

METR 4202: **Robotics** & Automation

Dr Surya Singh -- Lecture # 8

September 14, 2016

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[<http://metr4202.com>]

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Schedule of Events

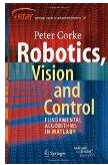
Week	Date	Lecture (W: 12:05-1:50, 50-N202)
1	27-Jul	Introduction
2	3-Aug	Representing Position & Orientation & State (Frames, Transformation Matrices & Affine Transformations)
3	10-Aug	Robot Kinematics Review (& <i>Ekka Day</i>)
4	17-Aug	Robot Inverse Kinematics & Kinetics
5	24-Aug	Robot Dynamics (Jacobians)
6	31-Aug	Robot Sensing: Perception & Linear Observers
7	7-Sep	Robot Sensing: Single View Geometry & Lines
8	14-Sep	Robot Sensing: Multiple View Geometry & Feature Detection
9	21-Sep	Probabilistic Robotics: Localization & SLAM
	28-Sep	<i>Study break</i>
10	5-Oct	Motion Planning
11	12-Oct	Planning & Control
12	19-Oct	State-Space Modelling
13	26-Oct	Shaping the Dynamic Response/LQR + Course Review



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Follow Along Reading:



[Robotics, Vision & Control](#)
by [Peter Corke](#)

Also online: [SpringerLink](#)

[UQ Library eBook:](#)
[364220144X](#)

Today

→ Sensing and Vision ←

- Multiple View Geometry
 - P. 47
 - Hartley & Zisserman:
Chapter 6: Camera Models
Chapter 7: Camera Matrix

- Localization
 - Chapter 6: Localization

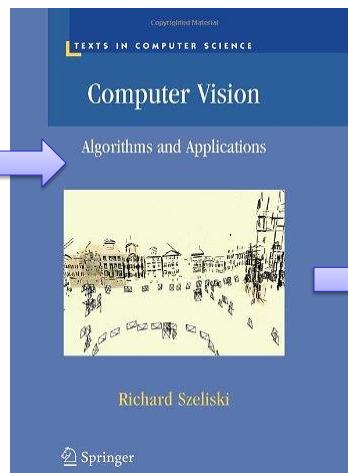
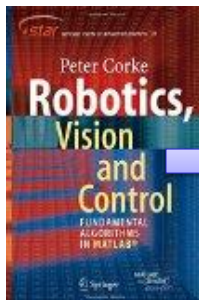
Next Time



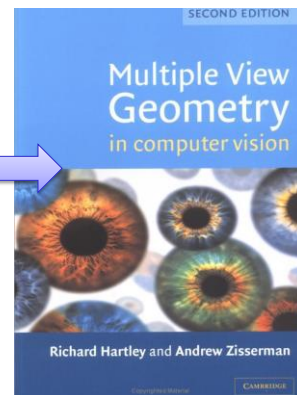
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Reference Material



[UQ Library/
SpringerLink](#)



[UQ Library
\(ePDF\)](#)



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Announcement: Monday Lab → Demo Thur | Fri

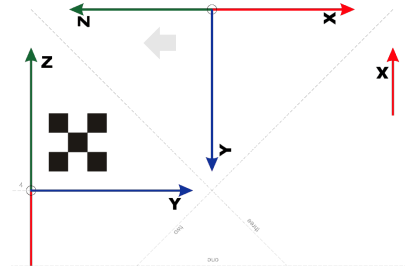
- Monday, October 3:
 - Queens Birthday Public Holiday
- “Makeup” Lab on Friday, October 7 from 4-6pm
- Monday Prac students may demo on Thursday (Oct 6) **or** Friday (Oct 7)
- Thursday Prac students to demo on Thursday (Oct 6)



SIFT / Corners for the {Frame} finder

To find the Frame, Consider:

- Structure
 - Corners
 - SIFT
 - ???
- Calibration Sequence
- Thought Experiment:
How do you make this traceable back to the {camera frame}



Camera matrix calibration

- Advantages:
 - very simple to formulate and solve
 - can recover $K [R | t]$ from M using QR decomposition [Golub & VanLoan 96]
- Disadvantages:
 - doesn't compute internal parameters
 - more unknowns than true degrees of freedom
 - need a separate camera matrix for each new view

From Szeliski, [Computer Vision: Algorithms and Applications](#)



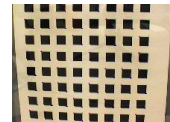
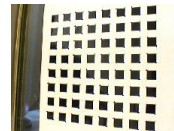
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Multi-plane calibration

- Use several images of planar target held at unknown orientations [Zhang 99]
 - Compute plane homographies
 - Solve for K -TK-1 from H_k 's
 - 1 plane if only f unknown
 - 2 planes if (f, u_c, v_c) unknown
 - 3+ planes for full K
 - Code available from Zhang and OpenCV

$$\begin{bmatrix} u_i \\ v_i \\ 1 \end{bmatrix} \sim K \begin{bmatrix} r_1 & r_2 & t \end{bmatrix} \begin{bmatrix} x_i \\ y_i \\ 1 \end{bmatrix} \sim HX$$



From Szeliski, [Computer Vision: Algorithms and Applications](#)



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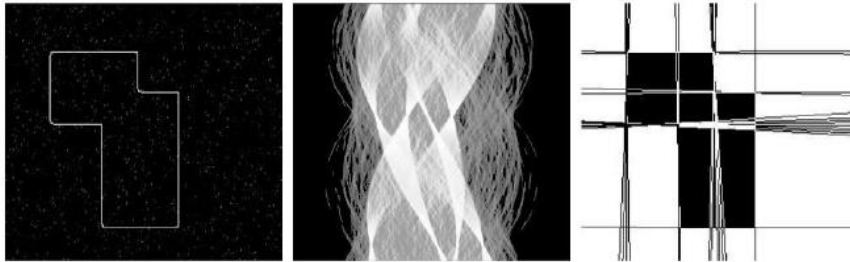
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Lines (Recap)

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Hough Transform



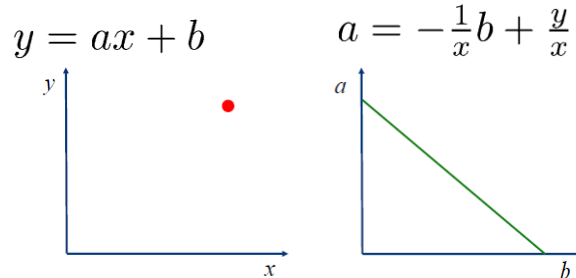
- Uses a voting mechanism
- Can be used for other lines and shapes
(not just straight lines)



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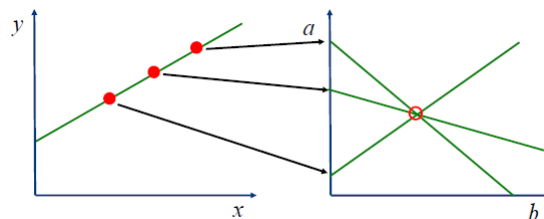
Hough Transform: Voting Space



- Count the number of lines that can go through a point and move it from the “x-y” plane to the “a-b” plane
- There is only a one-“infinite” number (a line!) of solutions (not a two-“infinite” set – a plane)



Hough Transform: Voting Space



- In practice, the polar form is often used

$$\rho = x \cos \theta + y \sin \theta$$
- This avoids problems with lines that are nearly vertical



Hough Transform: Algorithm

1. Quantize the parameter space appropriately.
2. Assume that each cell in the parameter space is an accumulator. Initialize all cells to zero.
3. For each point (x,y) in the (visual & range) image space, increment by 1 each of the accumulators that satisfy the equation.
4. Maxima in the accumulator array correspond to the parameters of model instances.



Line Detection – Hough Lines [1]

- A line in an image can be expressed as two variables:
 - Cartesian coordinate system: m,b
 - Polar coordinate system: r, θ
 - avoids problems with vert. lines

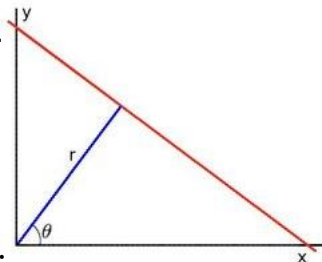
$$y = mx + b \rightarrow$$

$$y = \left(-\frac{\cos \theta}{\sin \theta} \right) x + \left(\frac{r}{\sin \theta} \right)$$

- For each point (x₁, y₁) we can write:

$$r = x_1 \cos \theta + y_1 \sin \theta$$

- Each pair (r, θ) represents a line that passes through (x₁, y₁)

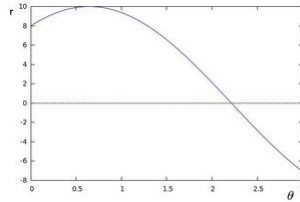


See also OpenCV documentation (cv::HoughLines)

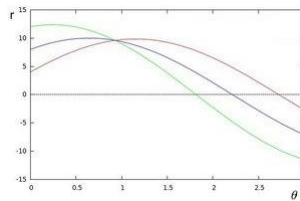


Line Detection – Hough Lines [2]

- Thus a given point gives a sinusoid



- Repeating for all points on the image



See also OpenCV documentation (`cv::HoughLines`)

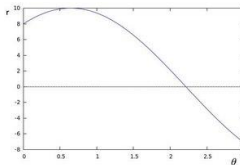


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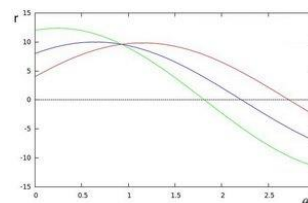
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Line Detection – Hough Lines [3]

- Thus a given point gives a sinusoid



- Repeating for all points on the image



- NOTE that an intersection of sinusoids represents **(a point)** represents **a line** in which pixel points lay.

➔ Thus, a line can be *detected* by finding the number of Intersections between curves

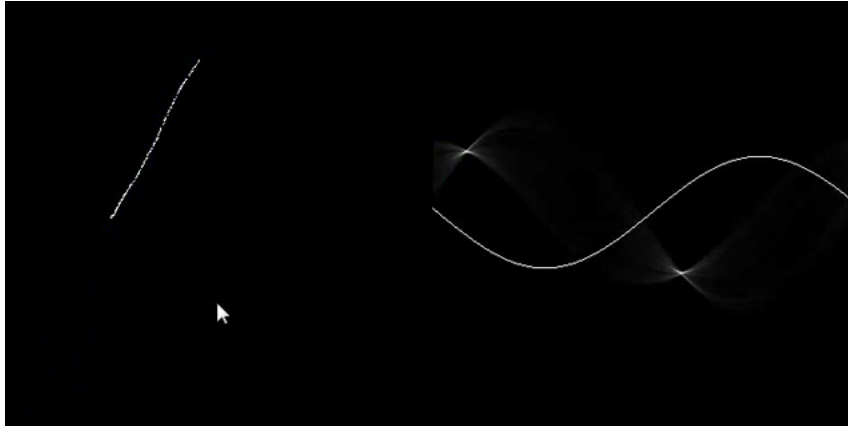
See also OpenCV documentation (`cv::HoughLines`)



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“Cool Robotics Share” -- Hough Transform



- http://www.activovision.com/octavi/doku.php?id=hough_transform



RANdom SAMple Consensus

1. Repeatedly select a small (minimal) subset of correspondences
 2. Estimate a solution (in this case a the line)
 3. Count the number of “inliers”, $|e| < \Theta$
(for LMS, estimate $\text{med}(|e|)$)
 4. Pick the *best* subset of inliers
 5. Find a complete least-squares solution
- Related to least median squares
 - See also:
MAPSAC (Maximum *A Posteriori* SAMple Consensus)



Feature Detection

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"A Rose By Any Other Name?"



3817674783595363744693879036326656034268381...
7674783595363744693879036326656034268

- SIFT

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How to get Matching Points? Features

- ~~Colour~~
- Corners
- Edges
- Lines
- Statistics on Edges: SIFT, SURF, ORB...

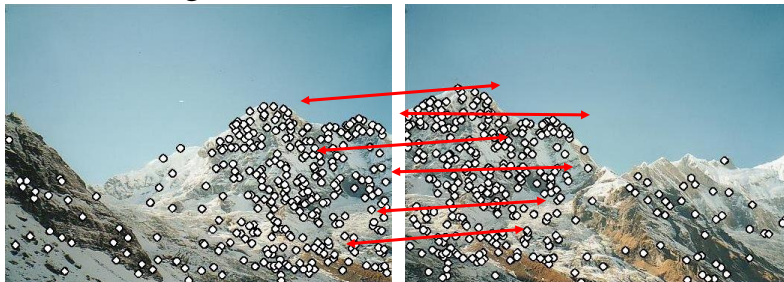
In OpenCV: The following detector types are supported:

- "FAST" – FastFeatureDetector
- "STAR" – StarFeatureDetector
- "SIFT" – SIFT (nonfree module)
- "SURF" – SURF (nonfree module)
- "ORB" – ORB
- "BRISK" – BRISK
- "MSER" – MSER
- "GFTT" – GoodFeaturesToTrackDetector
- "HARRIS" – GoodFeaturesToTrackDetector with Harris detector enabled
- "Dense" – DenseFeatureDetector
- "SimpleBlob" – SimpleBlobDetector



Why extract features?

- **Object detection**
- Robot Navigation
- Scene Recognition



- Steps:
 - Extract Features
 - Match Features

Adopted from S. Lazechnik, Gang Hua ([CS 558](#))



Why extract features? [2]

- Panorama stitching...
→ Step 3: Align images



Adopted from S. Lazebnik, Gang Hua ([CS 558](#))

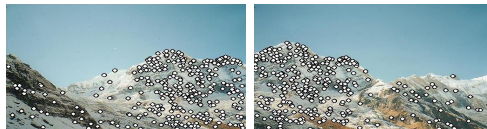


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Characteristics of good features

- Repeatability
 - The same feature can be found in several images despite geometric and photometric transformations
- Saliency
 - Each feature is distinctive
- Compactness and efficiency
 - Many fewer features than image pixels
- Locality
 - A feature occupies a relatively small area of the image; robust to clutter and occlusion



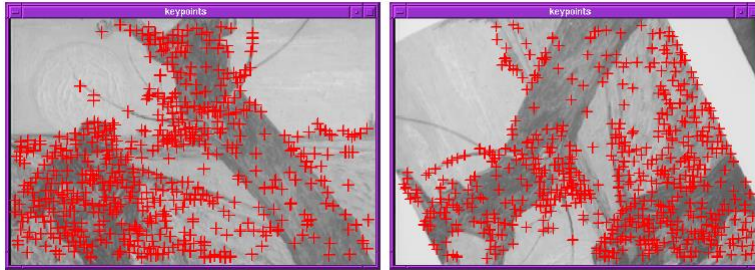
Adopted from S. Lazebnik, Gang Hua ([CS 558](#))



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Finding Corners



- Key property: in the region around a corner, image gradient has two or more dominant directions
- Corners are repeatable and distinctive

C.Harris and M.Stephens. "[A Combined Corner and Edge Detector.](#)" *Proceedings of the 4th Alvey Vision Conference*: pages 147—151, 1988.

Adopted from S. Lazebnik, Gang Hua ([CS 558](#))

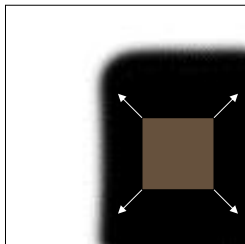


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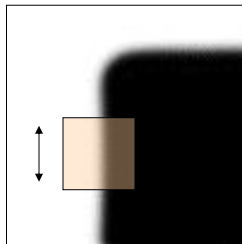
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Corner Detection: Basic Idea

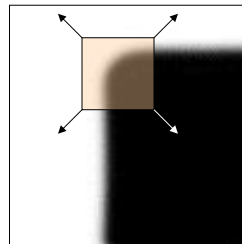
- Look through a window
- Shifting a window in any direction should give a large change in intensity



“flat” region:
no change in
all directions



“edge”:
no change along
the edge direction



“corner”:
significant change
in all directions

Source: A. Efros



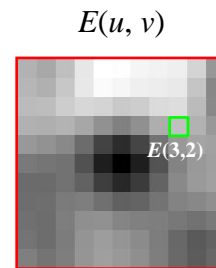
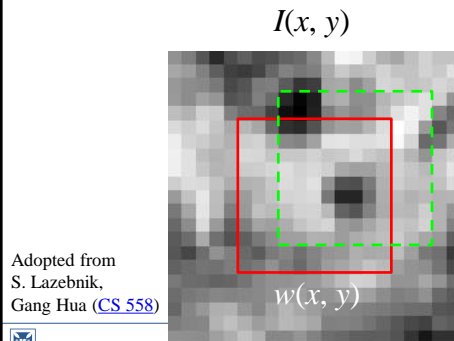
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Corner Detection: Mathematics

Change in appearance of window $w(x,y)$
for the shift $[u,v]$:

$$E(u,v) = \sum_{x,y} w(x,y) [I(x+u, y+v) - I(x,y)]^2$$

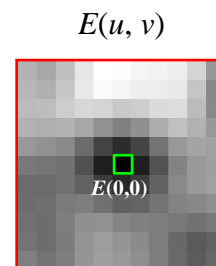
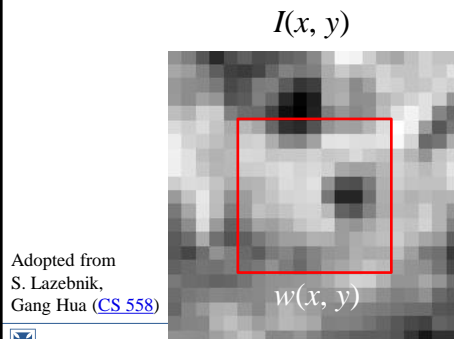


Adopted from
S. Lazebnik,
Gang Hua ([CS 558](#))

Corner Detection: Mathematics

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S. Lazebnik,
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Corner Detection: Mathematics

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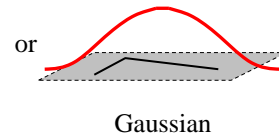
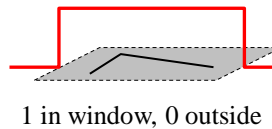
$$E(u, v) = \sum_{x,y} w(x, y) [I(x+u, y+v) - I(x, y)]^2$$

Window
function

Shifted
intensity

Intensity

Window function $w(x,y) =$



Adopted from
S. Lazebnik,
Gang Hua ([CS 558](#))

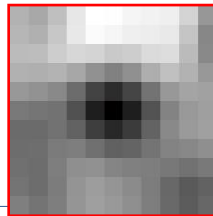
Corner Detection: Mathematics

Change in appearance of window $w(x,y)$
for the shift $[u,v]$:

$$E(u, v) = \sum_{x,y} w(x, y) [I(x+u, y+v) - I(x, y)]^2$$

We want to find out how this function behaves for small shifts

$E(u, v)$



Adopted from
S. Lazebnik,
Gang Hua ([CS 558](#))

Corner Detection: Mathematics

Change in appearance of window $w(x,y)$
for the shift $[u,v]$:

$$E(u,v) = \sum_{x,y} w(x,y) [I(x+u, y+v) - I(x,y)]^2$$

We want to find out how this function behaves for small shifts

$$E(u,v) \approx E(0,0) + [u \ v] \begin{bmatrix} E_u(0,0) \\ E_v(0,0) \end{bmatrix} + \frac{1}{2} [u \ v] \begin{bmatrix} E_{uu}(0,0) & E_{uv}(0,0) \\ E_{vu}(0,0) & E_{vv}(0,0) \end{bmatrix} \begin{bmatrix} u \\ v \end{bmatrix}$$

Local quadratic approximation of $E(u,v)$ in the neighborhood of $(0,0)$ is given by the *second-order Taylor expansion*:

Adopted from
S. Lazebnik,
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Corner Detection: Mathematics

$$E(u,v) = \sum_{x,y} w(x,y) [I(x+u, y+v) - I(x,y)]^2$$

Second-order Taylor expansion of $E(u,v)$ about $(0,0)$:

$$E(u,v) \approx E(0,0) + [u \ v] \begin{bmatrix} E_u(0,0) \\ E_v(0,0) \end{bmatrix} + \frac{1}{2} [u \ v] \begin{bmatrix} E_{uu}(0,0) & E_{uv}(0,0) \\ E_{vu}(0,0) & E_{vv}(0,0) \end{bmatrix} \begin{bmatrix} u \\ v \end{bmatrix}$$

$$E_u(u,v) = \sum_{x,y} 2w(x,y) [I(x+u, y+v) - I(x,y)] I_x(x+u, y+v)$$

$$E_{uu}(u,v) = \sum_{x,y} 2w(x,y) I_x(x+u, y+v) I_x(x+u, y+v) \\ + \sum_{x,y} 2w(x,y) [I(x+u, y+v) - I(x,y)] I_{xx}(x+u, y+v)$$

$$E_{uv}(u,v) = \sum_{x,y} 2w(x,y) I_y(x+u, y+v) I_x(x+u, y+v) \\ + \sum_{x,y} 2w(x,y) [I(x+u, y+v) - I(x,y)] I_{xy}(x+u, y+v)$$

Adopted from
S. Lazebnik,
Gang Hua ([CS 558](#))



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Corner Detection: Mathematics

$$E(u, v) = \sum_{x, y} w(x, y) [I(x+u, y+v) - I(x, y)]^2$$

Second-order Taylor expansion of $E(u, v)$ about $(0, 0)$:

$$E(u, v) \approx [u \ v] \begin{bmatrix} \sum_{x, y} w(x, y) I_x^2(x, y) & \sum_{x, y} w(x, y) I_x(x, y) I_y(x, y) \\ \sum_{x, y} w(x, y) I_x(x, y) I_y(x, y) & \sum_{x, y} w(x, y) I_y^2(x, y) \end{bmatrix} \begin{bmatrix} u \\ v \end{bmatrix}$$

$$E(0, 0) = 0$$

$$E_u(0, 0) = 0$$

$$E_v(0, 0) = 0$$

$$E_{uu}(0, 0) = \sum_{x, y} 2w(x, y) I_x(x, y) I_x(x, y)$$

$$E_{vv}(0, 0) = \sum_{x, y} 2w(x, y) I_y(x, y) I_y(x, y)$$

$$E_{uv}(0, 0) = \sum_{x, y} 2w(x, y) I_x(x, y) I_y(x, y)$$

Adopted from
S. Lazebnik,
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Harris detector: Steps

- Compute Gaussian derivatives at each pixel
- Compute second moment matrix M in a Gaussian window around each pixel
- Compute corner response function R
- Threshold R
- Find local maxima of response function (nonmaximum suppression)

C.Harris and M.Stephens. "[A Combined Corner and Edge Detector.](#)"
Proceedings of the 4th Alvey Vision Conference: pages 147—151, 1988.

Adopted from
S. Lazebnik,
Gang Hua ([CS 558](#))



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Harris Detector: Steps



Adopted from S. Lazebnik, Gang Hua ([CS 558](#))

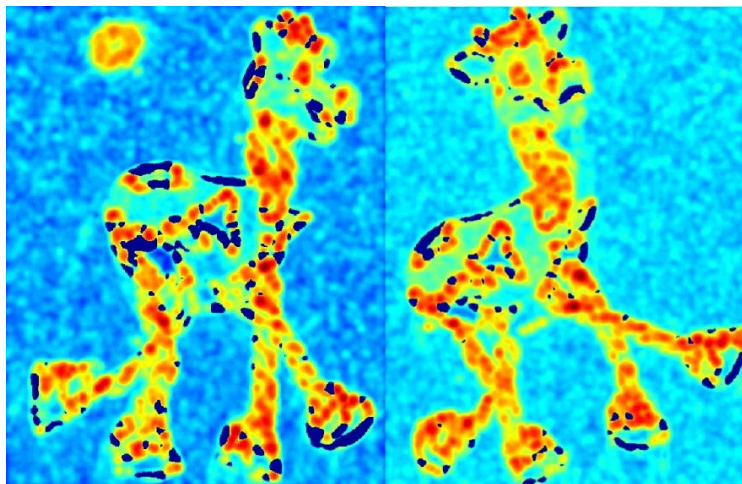


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Harris Detector: Steps

Compute corner response R



Adopted from S. Lazebnik, Gang Hua ([CS 558](#))



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Harris Detector: Steps

Find points with large corner response: $R > \text{threshold}$



Adopted from S. Lazebnik, Gang Hua ([CS 558](#))

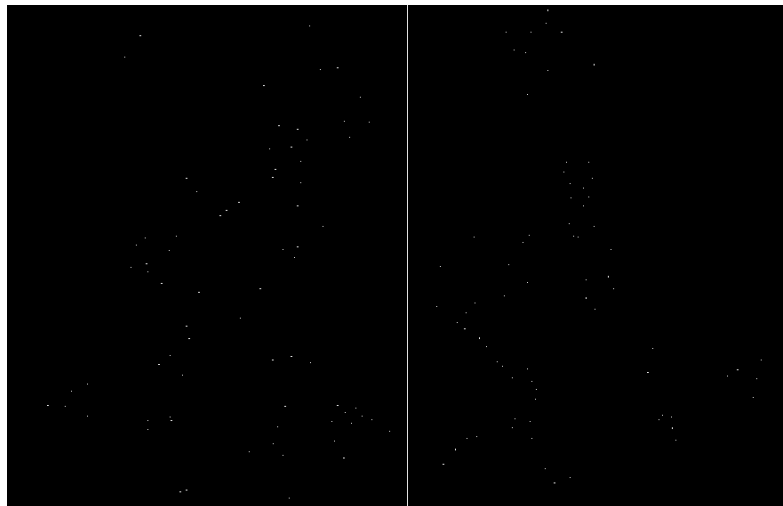


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Harris Detector: Steps

Take only the points of local maxima of R



Adopted from S. Lazebnik, Gang Hua ([CS 558](#))



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Harris Detector: Steps



Adopted from S. Lazebnik, Gang Hua ([CS 558](#))



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Invariance and covariance

- We want corner locations to be invariant to photometric transformations and covariant to geometric transformations
 - Invariance: image is transformed and corner locations do not change
 - Covariance: if we have two transformed versions of the same image, features should be detected in corresponding locations



Adopted from S. Lazebnik, Gang Hua ([CS 558](#))



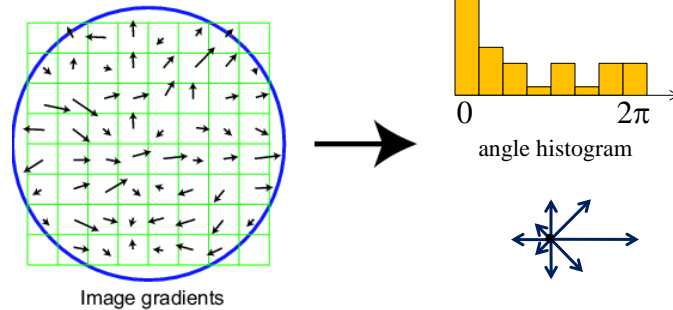
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Scale Invariant Feature Transform

Basic idea:

- Take 16x16 square window around detected feature
- Compute edge orientation (angle of the gradient - 90°) for each pixel
- Throw out weak edges (threshold gradient magnitude)
- Create histogram of surviving edge orientations



Adapted from slide by David Lowe



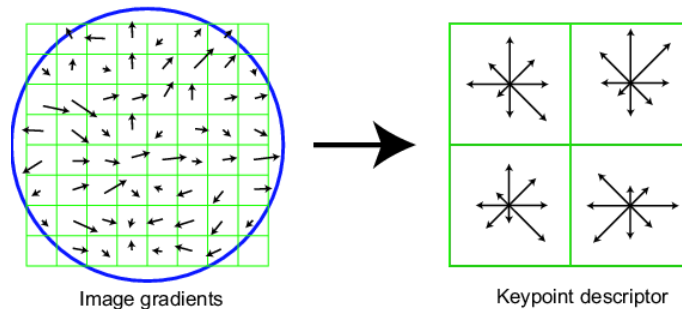
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SIFT descriptor

Full version

- Divide the 16x16 window into a 4x4 grid of cells (2x2 case shown below)
- Compute an orientation histogram for each cell
- 16 cells * 8 orientations = 128 dimensional descriptor



Adapted from slide by David Lowe

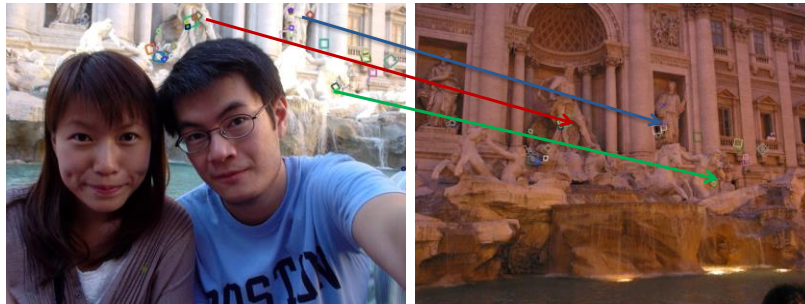


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Properties of SIFT

- Extraordinarily robust matching technique
 - Can handle changes in viewpoint
 - Up to about 60 degree out of plane rotation
 - Can handle significant changes in illumination
 - Sometimes even day vs. night (below)
 - Fast and efficient—can run in real time
 - Lots of code available
 - http://people.csail.mit.edu/albert/ladypack/wiki/index.php/Known_implementations_of_SIFT



From David Lowe and Szeliski, [Computer Vision: Algorithms and Applications](#)



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Feature matching

- Given a feature in I_1 , how to find the best match in I_2 ?
 1. Define distance function that compares two descriptors
 2. Test all the features in I_2 , find the one with min distance

From Szeliski, [Computer Vision: Algorithms and Applications](#)



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Feature distance

- How to define the difference between two features f_1, f_2 ?
 - Simple approach is $SSD(f_1, f_2)$
 - sum of square differences between entries of the two descriptors
 - can give good scores to very ambiguous (bad) matches



I_1

I_2

From Szeliski, [Computer Vision: Algorithms and Applications](#)



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Feature distance

- How to define the difference between two features f_1, f_2 ?
 - Better approach: ratio distance = $SSD(f_1, f_2) / SSD(f_1, f_2')$
 - f_2 is best SSD match to f_1 in I_2
 - f_2' is 2nd best SSD match to f_1 in I_2
 - gives small values for ambiguous matches



I_1

I_2

From Szeliski, [Computer Vision: Algorithms and Applications](#)

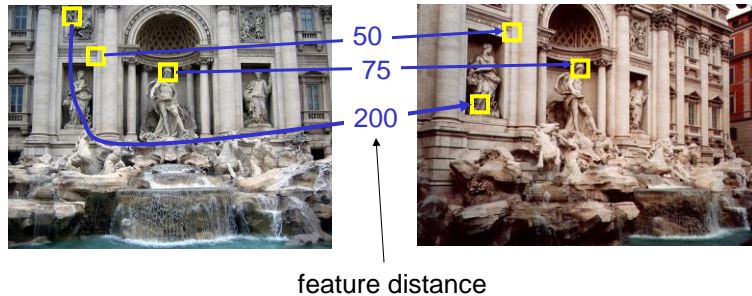


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Evaluating the results

- How can we measure the performance of a feature matcher?



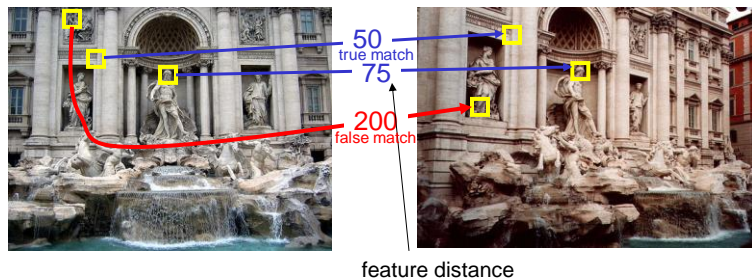
From Szeliski, [Computer Vision: Algorithms and Applications](#)



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True/false positives



- The distance threshold affects performance
 - True positives = # of detected matches that are correct
 - Suppose we want to maximize these—how to choose threshold?
 - False positives = # of detected matches that are incorrect
 - Suppose we want to minimize these—how to choose threshold?

From Szeliski, [Computer Vision: Algorithms and Applications](#)



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Levenberg-Marquardt

- Iterative non-linear least squares [Press'92]
 - Linearize measurement equations

$$\hat{u}_i = f(\mathbf{m}, \mathbf{x}_i) + \frac{\partial f}{\partial \mathbf{m}} \Delta \mathbf{m}$$

$$\hat{v}_i = g(\mathbf{m}, \mathbf{x}_i) + \frac{\partial g}{\partial \mathbf{m}} \Delta \mathbf{m}$$

- Substitute into log-likelihood equation:
quadratic cost function in $\Delta \mathbf{m}$

$$\sum_i \sigma_i^{-2} (\hat{u}_i - u_i + \frac{\partial f}{\partial \mathbf{m}} \Delta \mathbf{m})^2 + \dots$$

From Szeliski, [Computer Vision: Algorithms and Applications](#)



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Levenberg-Marquardt

- What if it doesn't converge?
 - Multiply diagonal by $(1 + l)$, increase l until it does
 - Halve the step size $\Delta \mathbf{m}$ (my favorite)
 - Use line search
 - Other ideas?
- Uncertainty analysis: covariance $\mathbf{S} = \mathbf{A}^{-1}$
- Is maximum likelihood the best idea?
- How to start in vicinity of global minimum?

From Szeliski, [Computer Vision: Algorithms and Applications](#)



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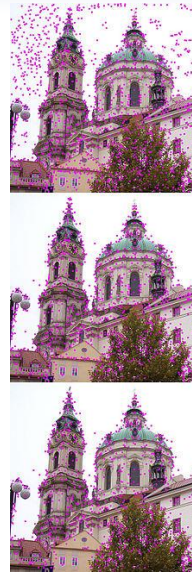
Feature Based Vision Extras

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Scale Invariant Feature Transforms

- Goal was to define an algorithm to describe an image with features
- This would enable a number of different applications:
 - Feature Matching
 - Object / Image Matching
 - Orientation / Homography Resolution



Wikipedia: Scale Invariant
Feature Transforms (2014)



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SIFT: Feature Definition

- SIFT features are defined as the local extrema in a Difference of Gaussian (D) Scale Pyramid.

$$D(x, y, \sigma) = L(x, y, k_i \sigma) - L(x, y, k_i \sigma)$$

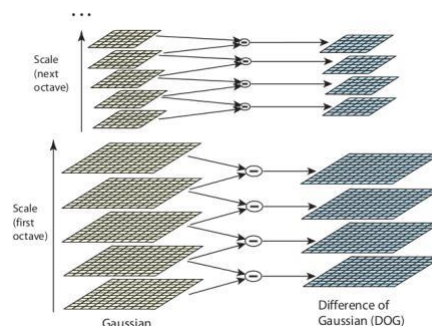
Where

$$L(x, y, k_i \sigma) = G(x, y, k \sigma) * I(x, y)$$



SIFT: Scale Pyramid

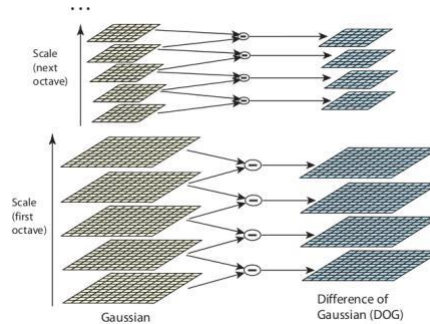
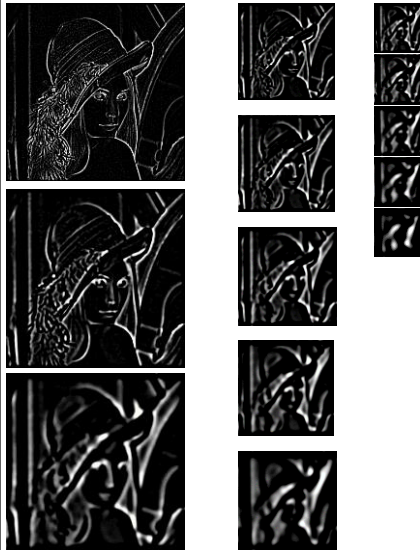
- D images are organised into a pyramid of progressively blurred images.
- Separated into octaves and scale levels per octave.
- Between octaves image is decimated by a factor of 2.



Lowe, D. G. (2004). Distinctive image features from scale-invariant keypoints. *International journal of computer vision*, 60(2), 91-110.



SIFT: Scale Pyramid



Lowe, D. G. (2004). Distinctive image features from scale-invariant keypoints. *International journal of computer vision*, 60(2), 91-110.

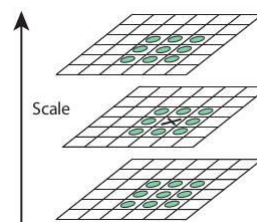


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SIFT: Feature Detection

- Each scale level in the image is evaluated for features.
- A feature is defined as a local maximum or minimum.
- For efficiency the 26 surrounding points are evaluated.



Lowe, D. G. (2004). Distinctive image features from scale-invariant keypoints. *International journal of computer vision*, 60(2), 91-110.



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SIFT: Feature Reduction

- Initial feature detection over detects features descriptive of the image.
- Initially remove features with low contrast.
- Then evaluate features to remove any edge responses.

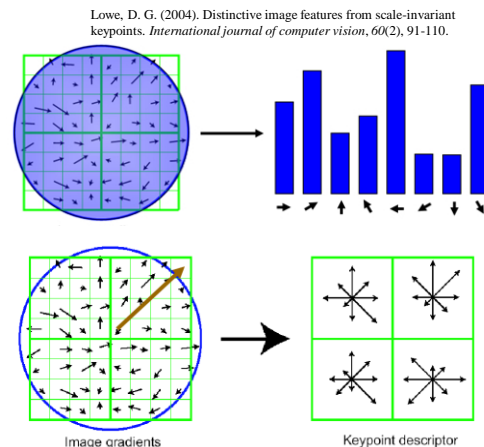


Wikipedia: Scale Invariant Feature Transforms (2014)



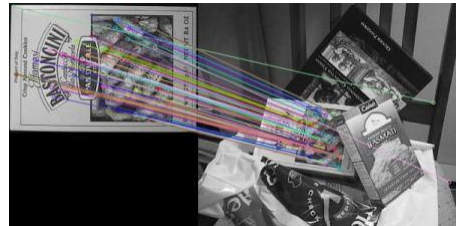
SIFT: Feature Description

- Features are described using the pixel gradients in a 16x16 square centring on the feature point.
- These gradients are then segmented into 4x4 boxes. An 8 bin orientation histogram is created to define the box.



SIFT: Feature Matching

- A match is defined as a pair of features with the closest Euclidian distance to each other.
- Matches above a threshold are culled to improve match.

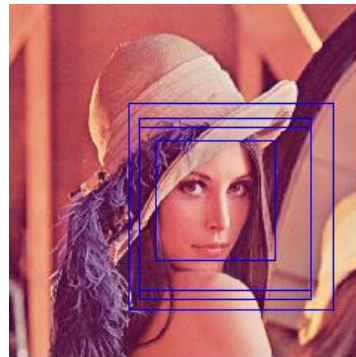


OpenCV: Feature Matching (2014)



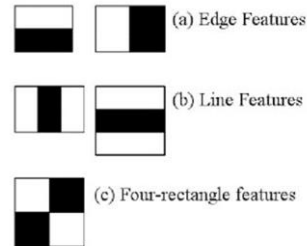
Boosted Cascade Haar-like Weak Classifiers

- Fast object detector designed primarily for use in face detection.
- Uses a cascade of weak classifiers to define object match.



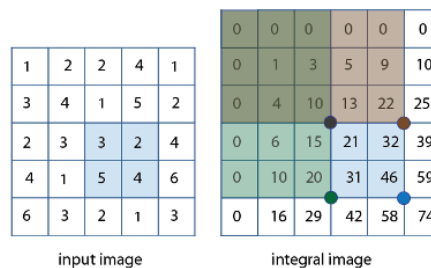
Viola Jones: Feature Definition

- Feature is classified as being the difference between the average intensity of two or more image sections.
- Can be any arithmetic combination of section values.



Viola Jones: Efficient Calculation of Features

- Fast calculation of the feature value is obtained by calculating the integral image.
- This leaves at most 4 sum operations to calculate a feature.



Viola Jones: Boosting

- Iteratively selects best classifier for detection.
- Assigns weights to each classifier to indicate likelihood of classifier indicating positive detection
- If the sum of the weights of positive classifier responses is above a threshold then there is a positive detection.

- Given example images $(x_1, y_1), \dots, (x_n, y_n)$ where $y_i = 0, 1$ for negative and positive examples respectively.
- Initialize weights $w_{1,i} = \frac{1}{2m}, \frac{1}{2l}$ for $y_i = 0, 1$ respectively, where m and l are the number of negatives and positives respectively.
- For $t = 1, \dots, T$:

1. Normalize the weights,

$$w_{t,i} \leftarrow \frac{w_{t,i}}{\sum_{j=1}^n w_{t,j}}$$

so that w_t is a probability distribution.

2. For each feature, j , train a classifier h_j which is restricted to using a single feature. The error is evaluated with respect to w_t , $\epsilon_j = \sum_i w_i |h_j(x_i) - y_i|$.
3. Choose the classifier, h_t , with the lowest error ϵ_t .
4. Update the weights:

$$w_{t+1,i} = w_{t,i} \beta_t^{1-e_i}$$

where $e_i = 0$ if example x_i is classified correctly, $e_i = 1$ otherwise, and $\beta_t = \frac{\epsilon_t}{1-\epsilon_t}$.

- The final strong classifier is:

$$h(x) = \begin{cases} 1 & \sum_{t=1}^T \alpha_t h_t(x) \geq \frac{1}{2} \sum_{t=1}^T \alpha_t \\ 0 & \text{otherwise} \end{cases}$$

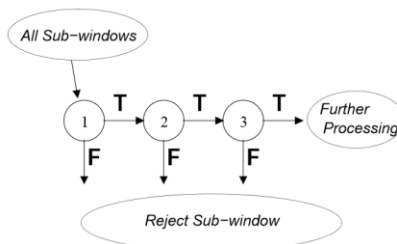
where $\alpha_t = \log \frac{1}{\beta_t}$

Viola, P., & Jones, M. (2001). Rapid object detection using a boosted cascade of simple features



Viola Jones: Boosted Cascades

- Effective boosted classifiers require a high number of weak classifiers.
- However, simple low count classifiers offer high rejection rate.
- Solution is to use cascaded classifiers.



Viola, P., & Jones, M. (2001). Rapid object detection using a boosted cascade of simple features

