Motion Planning

METR 4202: **Robotics** & Automation

Dr Surya Singh -- Lecture # 10

October 5, 2016

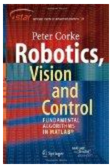
metr4202@itee.uq.edu.au

http://robotics.itee.uq.edu.au/~metr4202/ [http://**metr4202.com**]

Schedule of Events

Week	Date	Lecture (W: 12:05-1:50, 50-N202)
1	27-Jul	Introduction
2	3-Aug	Representing Position & Orientation & State (Frames, Transformation Matrices & Affine Transformations)
3	10-Aug	Robot Kinematics Review (& *Ekka Day*)
4	17-Aug	Robot Inverse Kinematics & Kinetics
5	24-Aug	Robot Dynamics (Jacobeans)
6	31-Aug	Robot Sensing: Perception & Linear Observers
7	7-Sep	Robot Sensing: Single View Geometry & Lines
8	14-Sep	Robot Sensing: Feature Detection
9	21-Sep	Robot Sensing: Multiple View Geometry
	28-Sep	*Study break*
10	**5-Oct**	**Motion Planning**
11	12-Oct	Probabilistic Robotics: Localization & SLAM
12	19-Oct	Probabilistic Robotics: Planning & Control
13	26-Oct	State-Space Automation (Shaping the Dynamic Response/LQR) + Course Review

Follow Along Reading:



Robotics, Vision & Control
by Peter Corke

Also online: SpringerLink

UQ Library eBook:
364220144X

Today

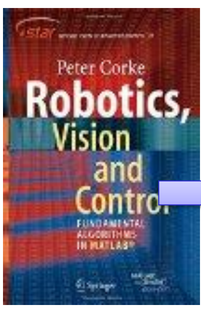
➔Planning & Control 🡐

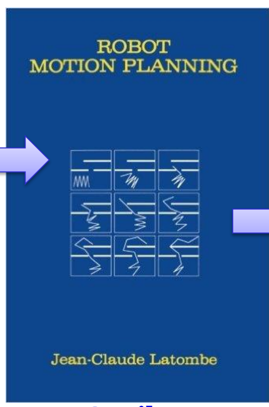
- Planning (**Global** Motion)
  - pp. 91-103
    (Yup! That's all Peter Corke has to say! Yet there is a Chapter [15] on Visual Servoing, a **local** motion method that can't handle obstacles).

- SLAM
  - pp. 123-4
    (§6.4-6.5)

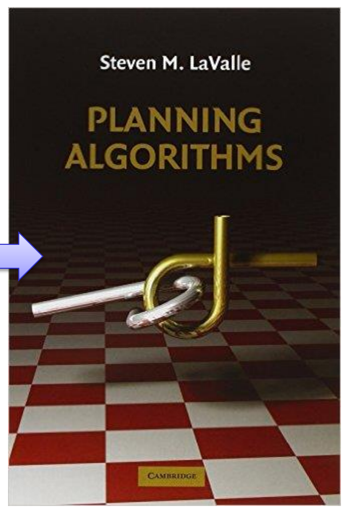
Next Time

Reference Material





UQ Library
(TJ211.4 .L38 1991)



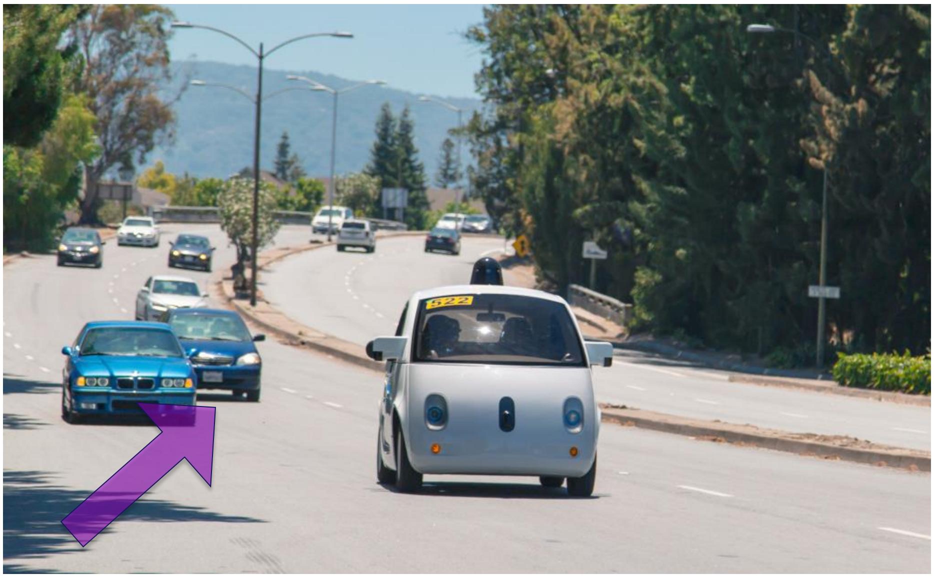
UQ Library / Online (PDF)

(Kinematic) Motion Planning

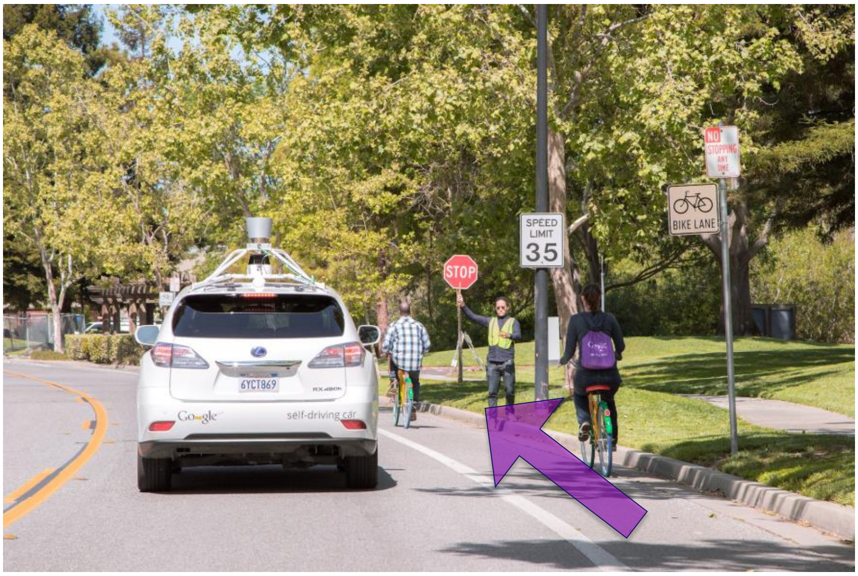
Motion Planning? Let's Get Moving…

Motion Planning? Let's Get Moving?

Motion Planning? The clutter can not be "ignored"



Motion Planning: Processing the Limits

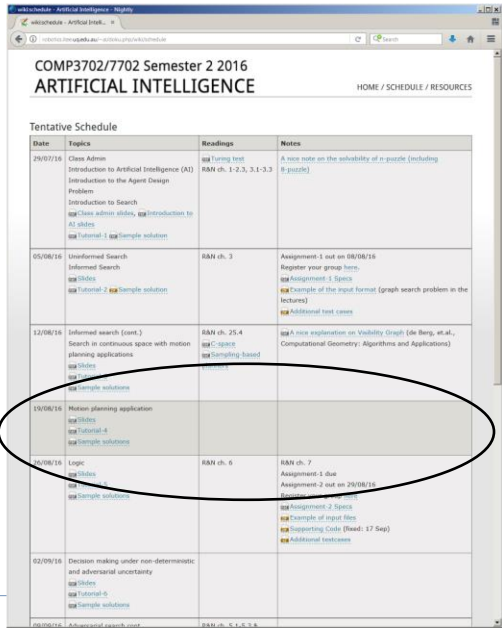


Path-Planning Approaches

- Roadmap
  Represent the connectivity of the free space by a network of 1-D curves
- Cell decomposition
  Decompose the free space into simple cells and represent the connectivity of the free space by the adjacency graph of these cells
- Potential field
  Define a function over the free space that has a global minimum at the goal configuration and follow its steepest descent

Slide from Latombe, CS326A

See Also: http://robotics.itee.uq.edu.au/~ai/



COMP3702/7702 Semester 2 2016
ARTIFICIAL INTELLIGENCE

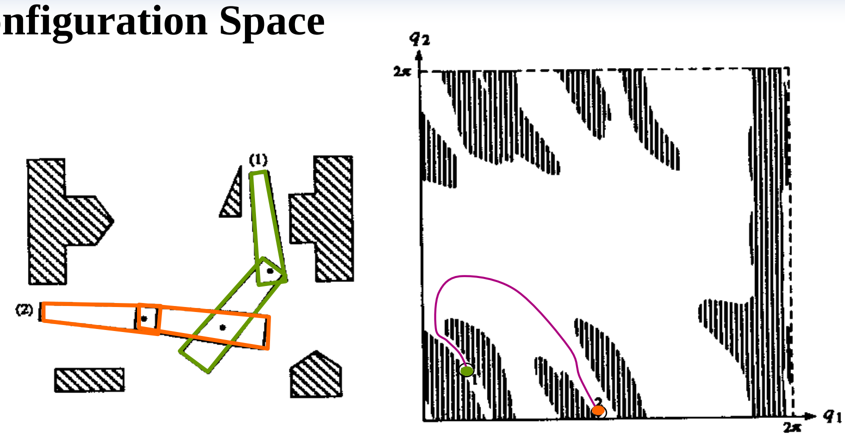
HOME / SCHEDULE / RESOURCES

Tentative Schedule

Date	Topics	Readings	Notes
29/07/16	Class Admin Introduction to Artificial Intelligence (AI) Introduction to the Agent Design Problem Introduction to Search Class admin slides, Introduction to AI slides Tutorial-1 Sample solution	Turing test R&N ch. 1-2.3, 3.1-3.3	A nice note on the solvability of n-puzzle (including 8-puzzle)
05/08/16	Uninformed Search Informed Search Slides Tutorial-2 Sample solution	R&N ch. 3	Assignment-1 out on 08/08/16 Register your group here. Assignment-1 Specs Example of the input format (graph search problem in the lectures) Additional test cases
12/08/16	Informed search (cont.) Search in continuous space with motion planning applications Slides [illegible] Sample solutions	R&N ch. 25.4 C-space Sampling-based [illegible]	A nice explanation on Visibility Graph (de Berg, et.al., Computational Geometry: Algorithms and Applications)
19/08/16	Motion planning application Slides Tutorial-4 Sample solutions		
26/08/16	Logic Slides [illegible] Sample solutions	R&N ch. 6	R&N ch. 7 Assignment-1 due Assignment-2 out on 29/08/16 Register [illegible] here Assignment-2 Specs Example of input files Supporting Code (fixed: 17 Sep) Additional testcases
02/09/16	Decision making under non-deterministic and adversarial uncertainty Slides Tutorial-6 Sample solutions		

External Configuration Is Important …

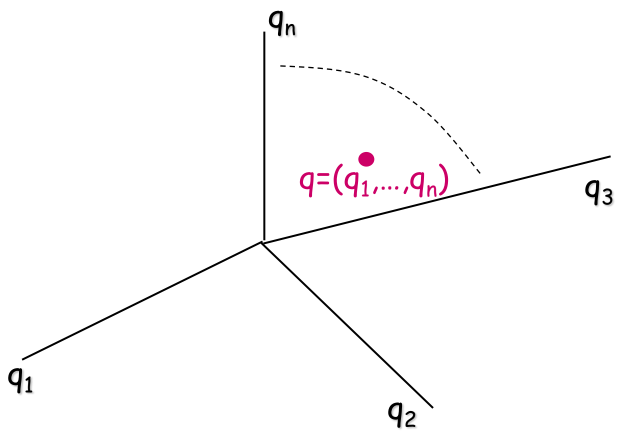
Configuration Space



- A robot configuration is a specification of the positions of all robot points relative to a fixed coordinate system
- Usually a configuration is expressed as a "vector" of position/orientation parameters

Slide from Latombe, CS326A

Motion Planning in C-Space

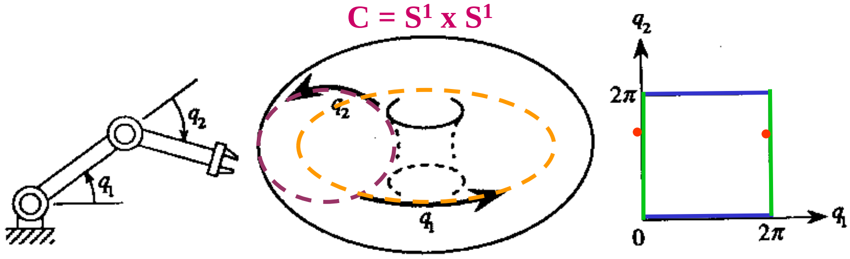


Slide from Latombe, CS326A

Configuration Space of a Robot

- Space of all its possible configurations
- But the topology of this space is usually not that of a Cartesian space

$C = S^1 \times S^1$



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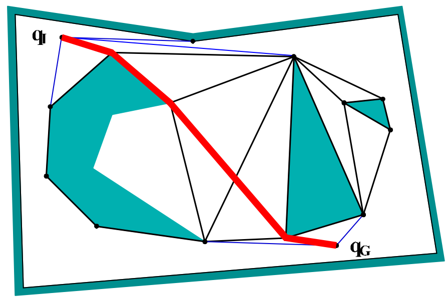
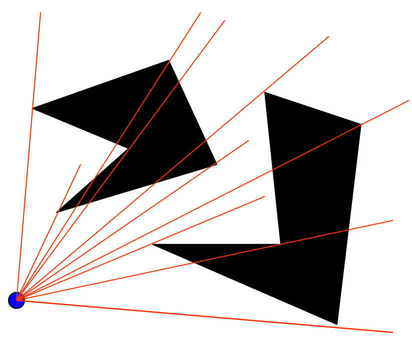
Geometric Planning Methods



- Several Geometric Methods:
  - Vertical (Trapezoidal) Cell Decomposition
  - **Roadmap Methods**
    - **Cell (Triangular) Decomposition**
    - Visibility Graphs
    - Veroni Graphs

Artwork from LaValle, Ch. 6

I. Rotational Sweep



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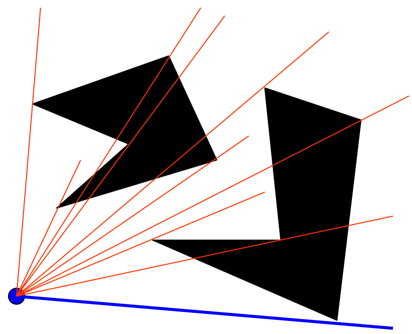
Rotational Sweep



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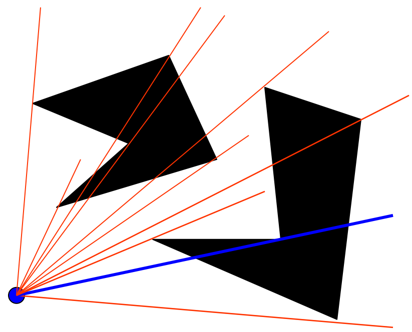
Rotational Sweep



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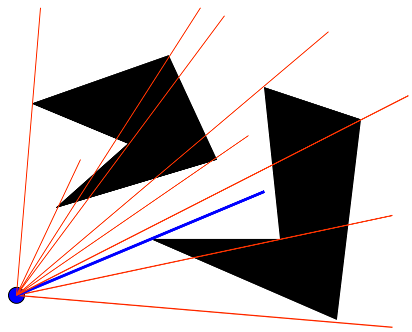
Rotational Sweep



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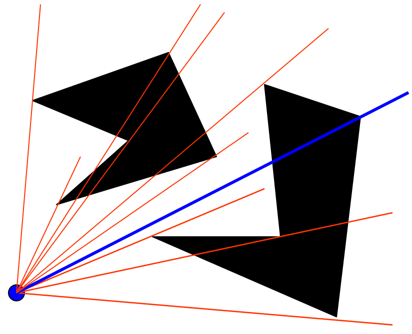
Rotational Sweep



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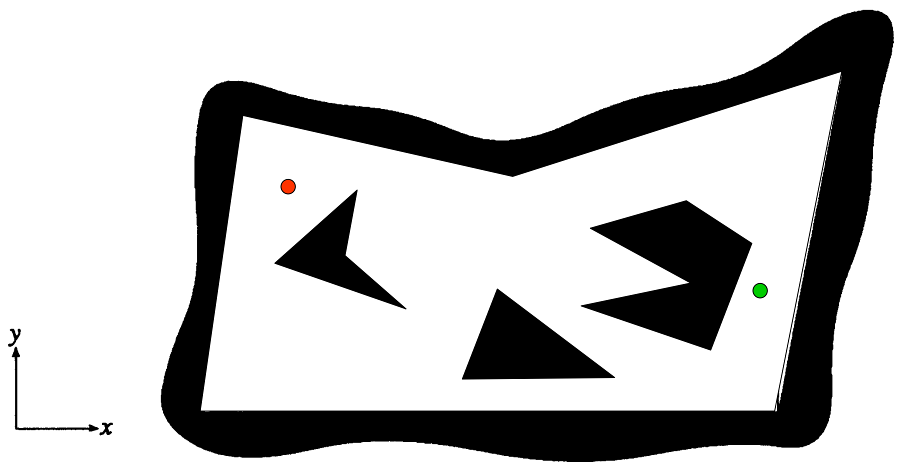
II. Cell-Decomposition Methods

Two classes of methods:

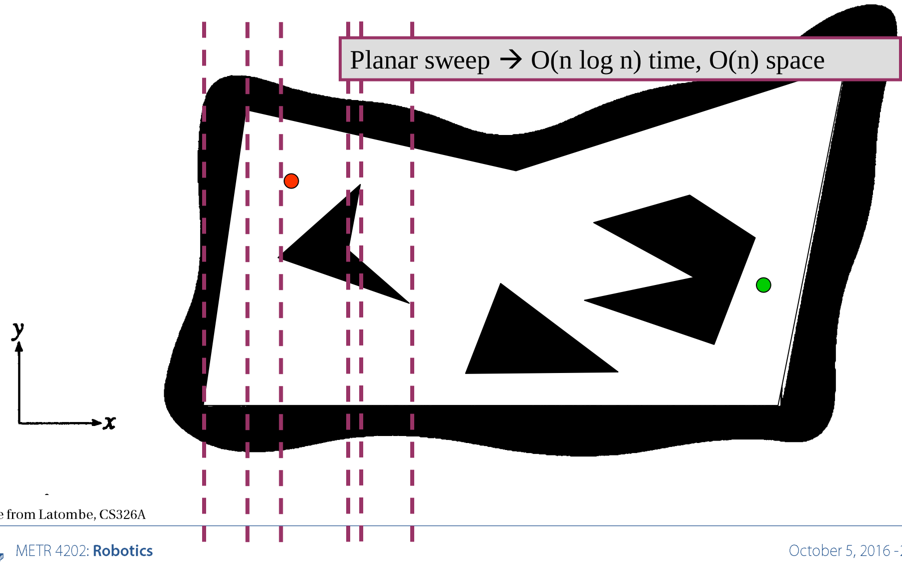
- Exact cell decomposition
  - The free space **F** is represented by a collection of non-overlapping cells whose union is exactly **F**
  - Example: trapezoidal decomposition
- Approximate cell decomposition
  - **F** is represented by a collection of non-overlapping cells whose union is contained in **F**
    Examples: quadtree, octree, 2n-tree

Trapezoidal decomposition



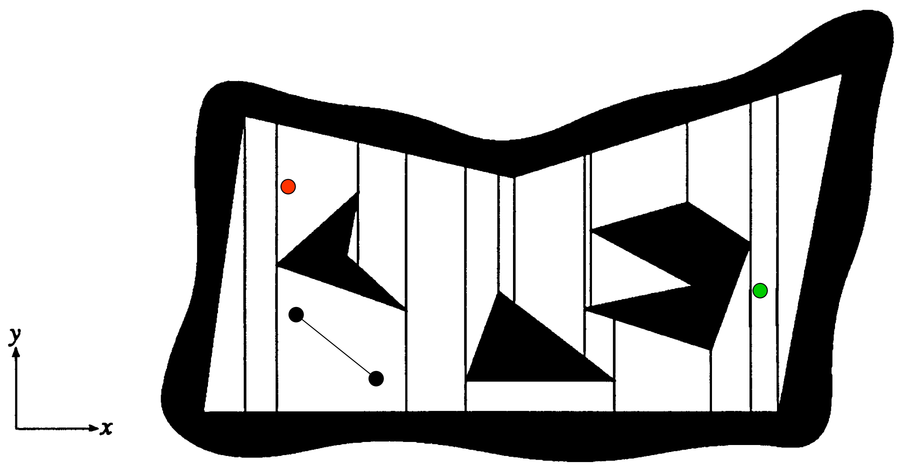
Slide from Latombe, CS326A

Trapezoidal decomposition



Slide from Latombe, CS326A

Trapezoidal decomposition



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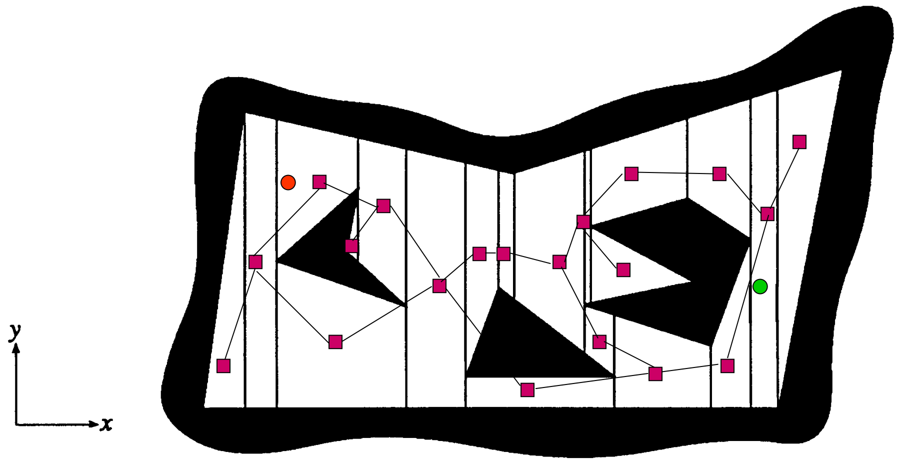
Trapezoidal decomposition



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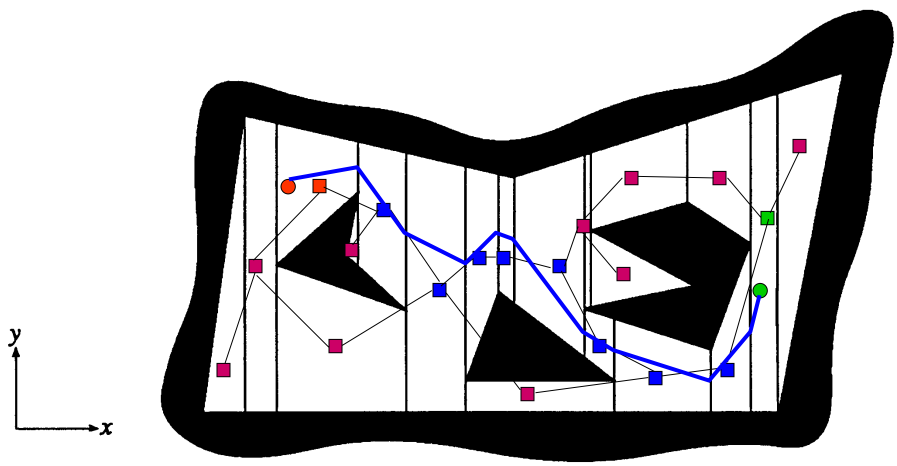
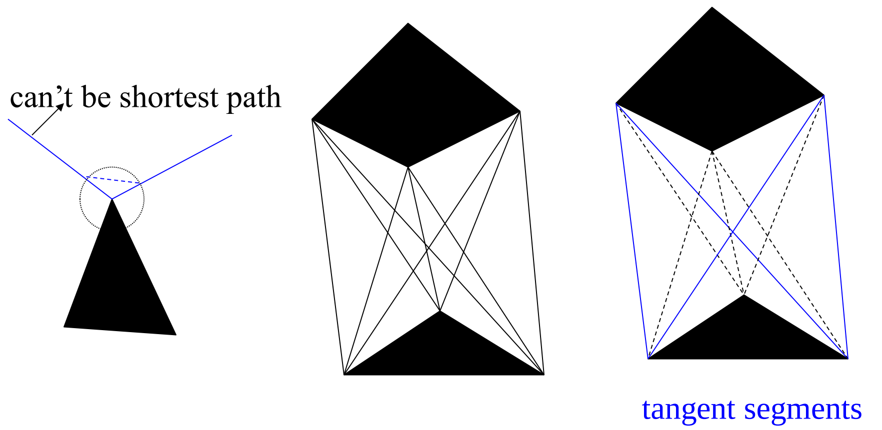
Trapezoidal decomposition



Slide from Latombe, CS326A

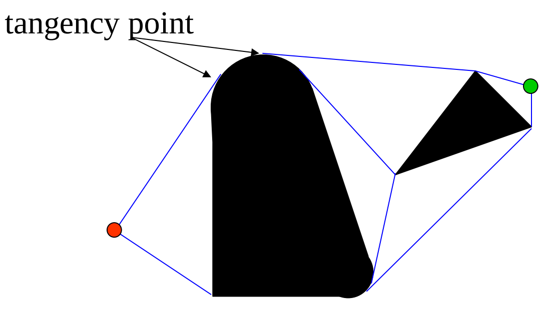
II.Visibility Graph



→ Eliminate concave obstacle vertices

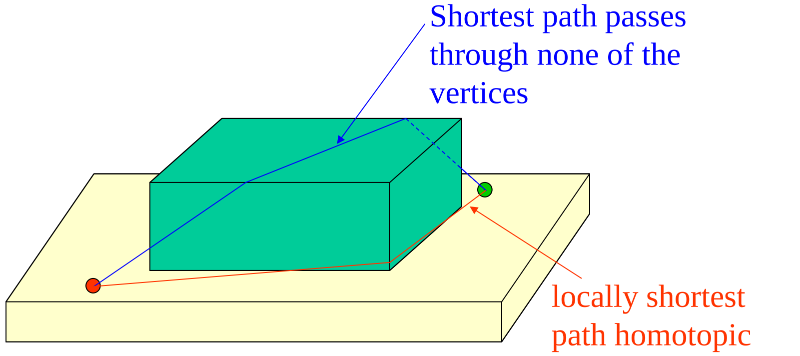
Slide from Latombe, CS326A

Generalized (Reduced) -- Visibility Graph



Slide from Latombe, CS326A

Three-Dimensional Space

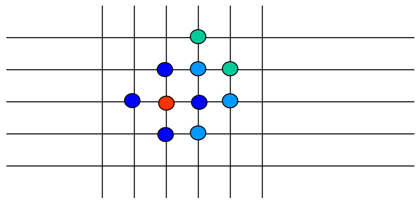


Computing the shortest collision-free path in a polyhedral space is NP-hard

Slide from Latombe, CS326A

Sketch of Grid Algorithm (with best-first search)

- Place regular grid G over space
- Search G using best-first search algorithm with potential as heuristic function



Slide from Latombe, CS326A

Simple Algorithm (for Visibility Graphs)

- Install all obstacles vertices in VG, plus the start and goal positions
- For every pair of nodes u, v in VG

    If segment(u,v) is an obstacle edge then

        insert (u,v) into VG

    else

    for every obstacle edge e

        if segment(u,v) intersects e

            then go up to segment

    insert (u,v) into VG
- Search VG using A*

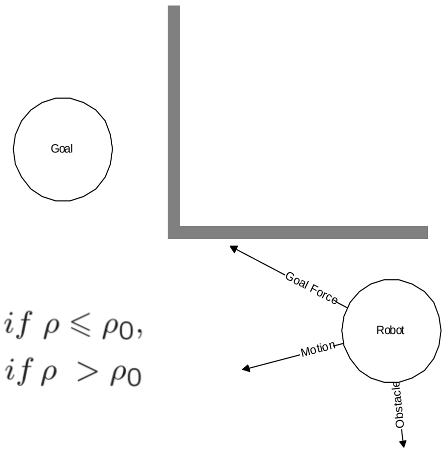
Slide based on Latombe, CS326A

III. Potential Field Methods

- Approach initially proposed for real-time collision avoidance [Khatib, 86]

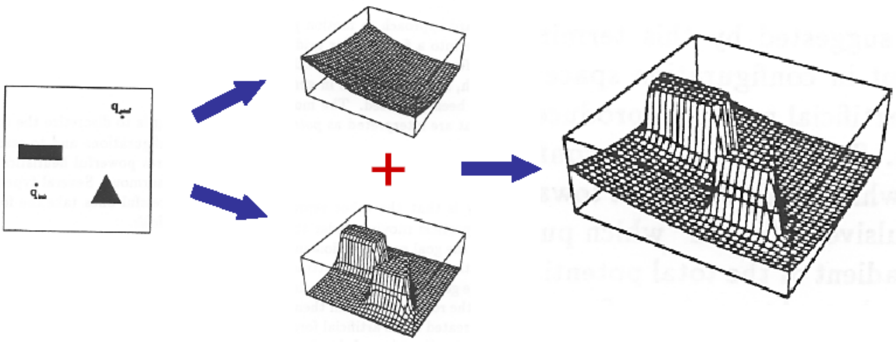
$$F_{Goal} = -k_p(x - x_{Goal})$$

$$F_{Obstacle} = \begin{cases} \eta \left(\frac{1}{\rho} - \frac{1}{\rho_0}\right) \frac{1}{\rho^2}\frac{\partial \rho}{\partial x} & if\ \rho \leqslant \rho_0, \\ 0 & if\ \rho > \rho_0 \end{cases}$$



Slide based on Latombe, CS326A

Attractive and Repulsive fields



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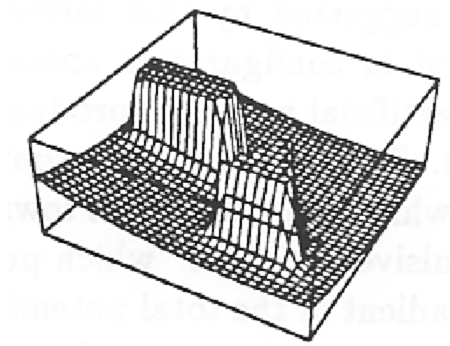
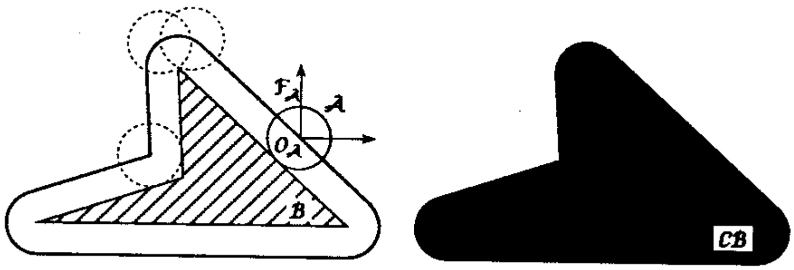
Local-Minimum Issue



- Perform best-first search (possibility of combining with approximate cell decomposition)
- Alternate descents and random walks
- Use local-minimum-free potential (navigation function)

Slide from Latombe, CS326A

Disc Robot in 2-D Workspace



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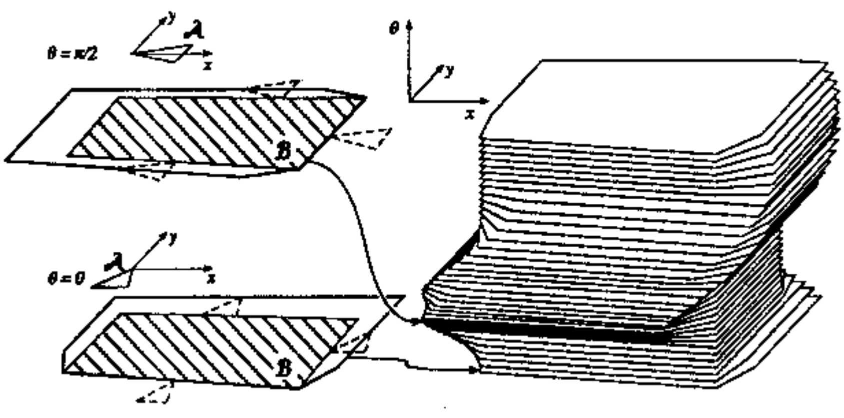
Rigid Robot Translating and Rotating in 2-D



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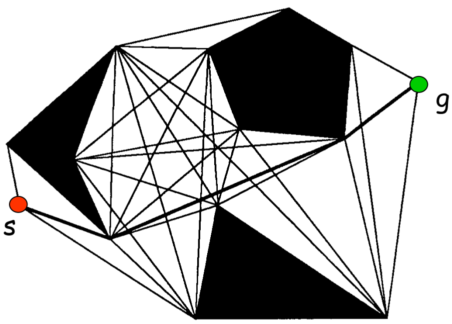
IV. Roadmap Methods

- **Visibility graph**
- **Voronoi diagram**
- Silhouette
  First complete general method that applies to spaces of any dimension and is singly exponential in # of dimensions [Canny, 87]
- **Probabilistic roadmaps (PRMS) and Rapidly-exploring Randomized Trees (RRTs)**

Slide from Latombe, CS326A

Roadmap Methods

- **Visibility graph**
  Introduced in the Shakey project at SRI in the late 60s. Can produce shortest paths in 2-D configuration spaces



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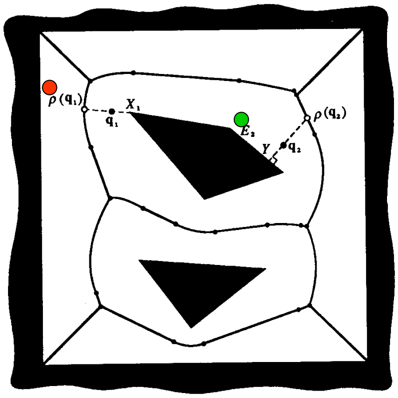
Roadmap Methods

- Voronoi diagram
  Introduced by Computational Geometry researchers. Generate paths that maximizes clearance.

  O(n log n) time
  O(n) space



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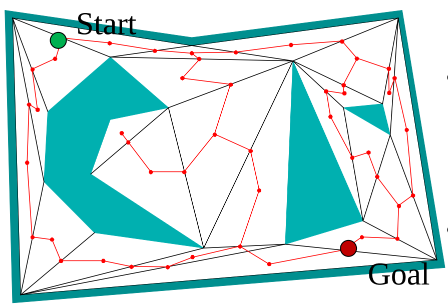
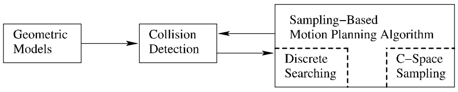
Limits of Geometric Planning Methods



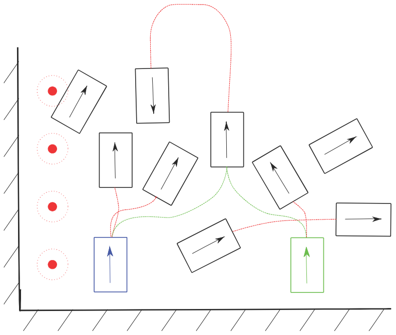
- How does this scale to high degrees of freedom?
- What about "dynamic constraints"?
- What about optimality?
- How to tie this to learning and optimization

Artwork from LaValle, Ch. 6

Sample-Based Motion Planning

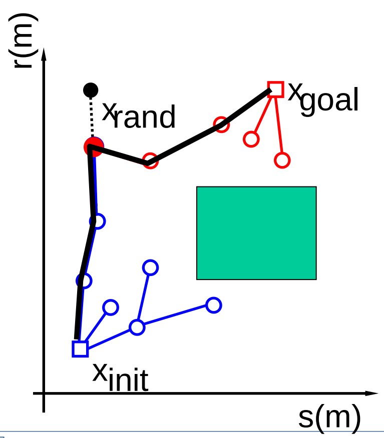


- PRMs
- RRTs

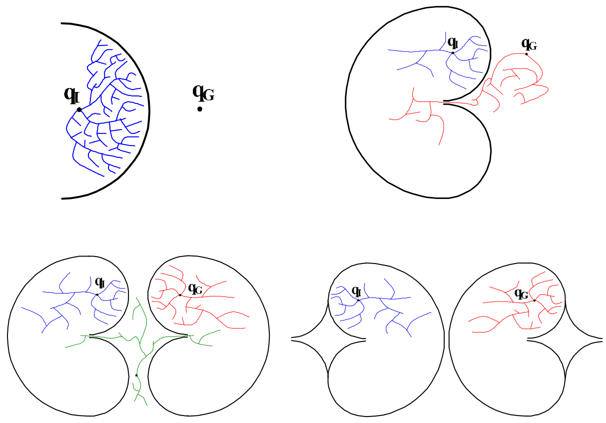


Artwork based on LaValle, Ch. 5

Rapidly Exploring Random Trees (RRT)

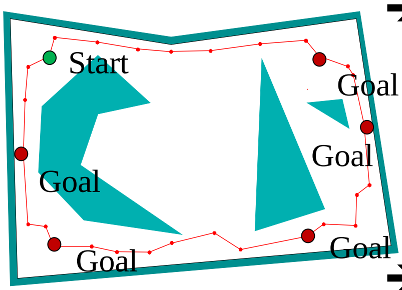


Sampling and the "Bug Trap" Problem



Artwork based on LaValle, Ch. 5

Multiple Points & Sequencing



- Sequencing
  - Determining the "best" order to go in

➔ Travelling Salesman Problem

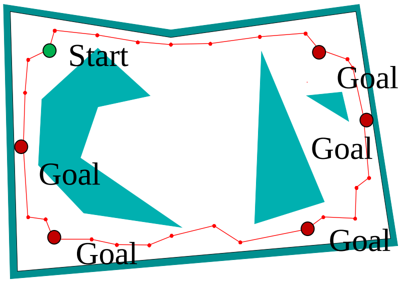
A salesman has to visit each city on a given list exactly once. In doing this, he **starts** from his home city and in the **end he has to return to his home** city. It is plausible for him to select the order in which he visits the cities so that the **total of the distances travelled** in his tour is as small as possible.

➔ Multi-Goal Problem

A salesman has to visit each city on a given list exactly once. In doing this, he **starts** from his home city and in the **end he has to return to his home** city. It is plausible for him to select the order in which he visits the cities so that the **total of the distances travelled** in his tour is as small as possible.

Artwork based on LaValle, Ch. 6

Travelling Salesman Problem

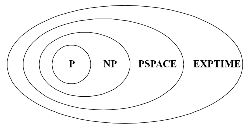


- Given a $n \times n$ distance matrix $\mathbf{C}=(c_{ij})$

- Minimize:

$$c(\pi) = \sum_{i=1}^{n} c_{i\pi(i)}$$

- Note that this problem is NP-Hard

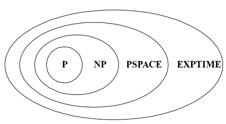


➔ BUT, Special Cases are Well-Solvable!

Artwork based on LaValle, Ch. 6

Travelling Salesman Problem [2]

- This problem is NP-Hard



➔ BUT,
Special Cases are
Well-Solvable!

For the Euclidean case
(where the points are on the 2D Euclidean plane) :

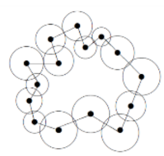
- The shortest TSP tour does not intersect itself, and thus geometry makes the problem somewhat easier.
- If all cities lie on the boundary of a convex polygon, the optimal tour is a cyclic walk along the boundary of the polygon (in clockwise or counterclockwise direction).

***The k*-line TSP**

- The a special case where the cities lie on *k* parallel (or almost parallel) lines in the Euclidean plane.
- EG: Fabrication of printed circuit boards
- Solvable in $\mathbf{O}(n^3)$ time by Dynamic Programming (Rote's algorithm)

The necklace TSP

- The special Euclidean TSP case where there exist *n* circles around the *n* cities such that every cycle intersects exactly two adjacent circles



Cool Robotics Share