Quiz

+ ~~Guest Lecture~~ (Postponed) on SLAM + Q&A

METR 4202: Advanced Control & **Robotics**

Dr Surya Singh -- Lecture # 9

September 23, 2015

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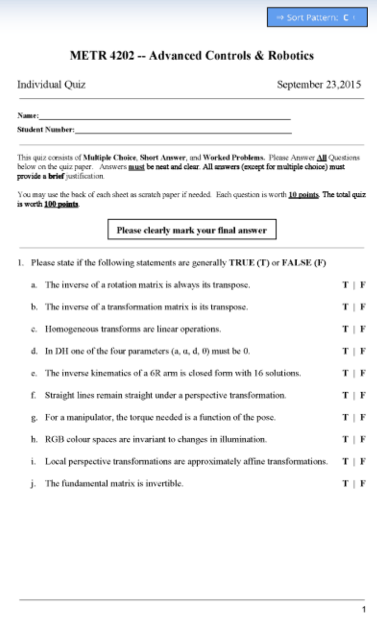
Schedule

Week	Date	Lecture (W: 12:05-1:50, 50-N201)
1	29-Jul	Introduction
2	5-Aug	Representing Position & Orientation & State (Frames, Transformation Matrices & Affine Transformations)
3	12-Aug	Robot Kinematics Review (& *Ekka Day*)
4	19-Aug	Robot Dynamics
5	26-Aug	Robot Sensing: Perception
6	2-Sep	Robot Sensing: Multiple View Geometry
7	9-Sep	Robot Sensing: Feature Detection (as Linear Observers)
8	16-Sep	Probabilistic Robotics: Localization
9	**23-Sep**	**Quiz** & ~~Guest Lecture~~ (Tabled) & SLAM
	30-Sep	*Study break*
10	7-Oct	Motion Planning
11	14-Oct	State-Space Modelling
12	21-Oct	Shaping the Dynamic Response
13	28-Oct	LQR + Course Review

Quiz

- 1 hour (1.5 × Tutor)
- 3 **Random** Sort Patterns
- 10 Questions



⇒ Sort Pattern: C

METR 4202 -- Advanced Controls & Robotics

Individual Quiz September 23,2015

Name:________________________

Student Number:________________________

This quiz consists of **Multiple Choice**, **Short Answer**, and **Worked Problems**. Please Answer **All** Questions below on the quiz paper. Answers **must be neat and clear**. **All answers (except for multiple choice) must provide a brief** justification.

You may use the back of each sheet as scratch paper if needed. Each question is worth **10 points**. **The total quiz is worth 100 points**.

Please clearly mark your final answer

1. Please state if the following statements are generally **TRUE (T)** or **FALSE (F)**

   a. The inverse of a rotation matrix is always its transpose. **T | F**

   b. The inverse of a transformation matrix is its transpose. **T | F**

   c. Homogeneous transforms are linear operations. **T | F**

   d. In DH one of the four parameters (a, α, d, θ) must be 0. **T | F**

   e. The inverse kinematics of a 6R arm is closed form with 16 solutions. **T | F**

   f. Straight lines remain straight under a perspective transformation. **T | F**

   g. For a manipulator, the torque needed is a function of the pose. **T | F**

   h. RGB colour spaces are invariant to changes in illumination. **T | F**

   i. Local perspective transformations are approximately affine transformations. **T | F**

   j. The fundamental matrix is invertible. **T | F**

1

SLAM!
(Better than SMAL!)

What is SLAM?

- SLAM asks the following question:

  Is it possible for an autonomous vehicle to start at an unknown location in an unknown environment and then to incrementally build a map of this environment while simultaneously using this map to compute vehicle location?

- SLAM has many indoor, outdoor, in-air and underwater applications for both manned and autonomous vehicles.

- Examples
  - Explore and return to starting point (Newman)
  - Learn trained paths to different goal locations
  - Traverse a region with complete coverage (eg, mine fields, lawns, reef monitoring)
  - …

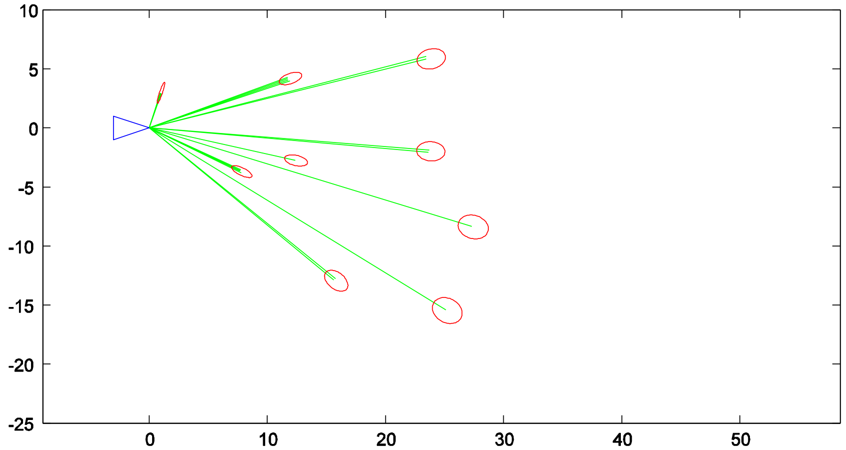
Components of SLAM

- Localisation
  - Determine pose given a priori map
- Mapping
  - Generate map when pose is accurately known from auxiliary source.
- SLAM
  - Define some arbitrary coordinate origin
  - Generate a map from on-board sensors
  - Compute pose from this map
  - Errors in map and in pose estimate are dependent.

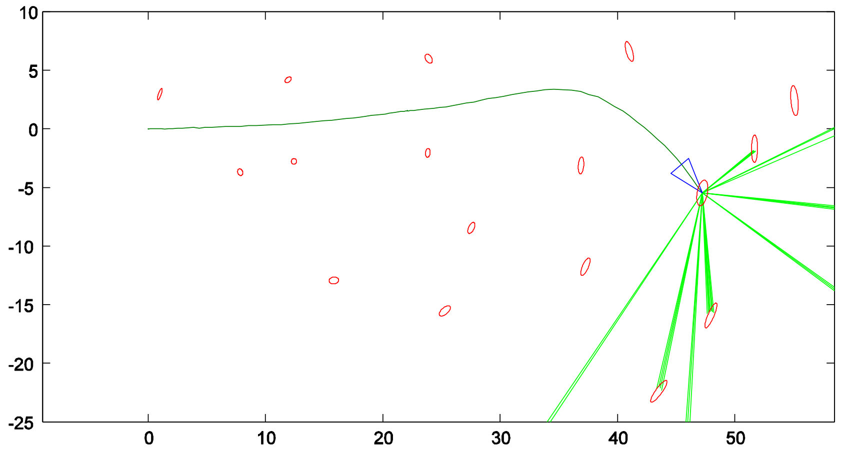
Basic SLAM Operation

Example: SLAM in Victoria Park

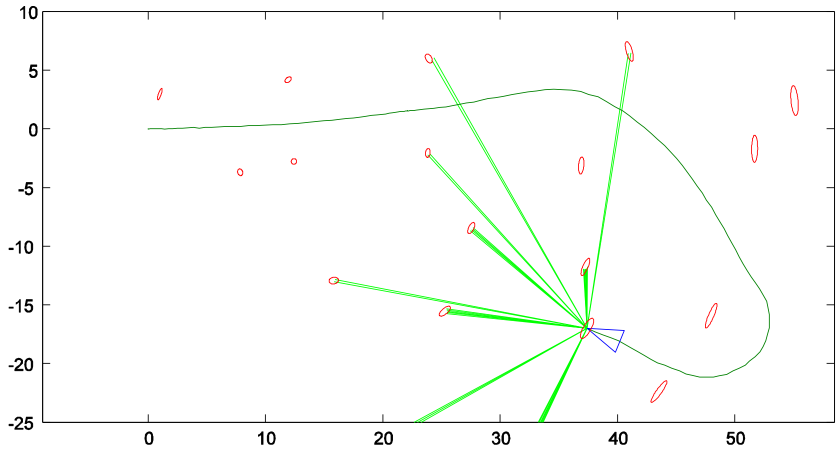
Basic SLAM Operation



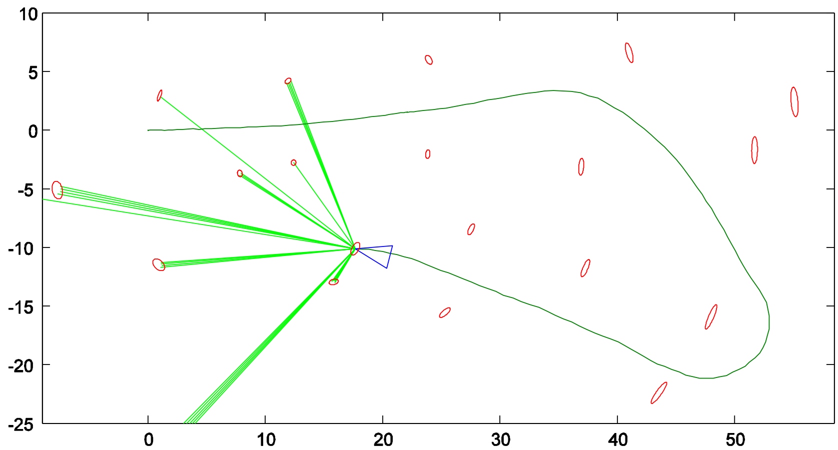
Basic SLAM Operation



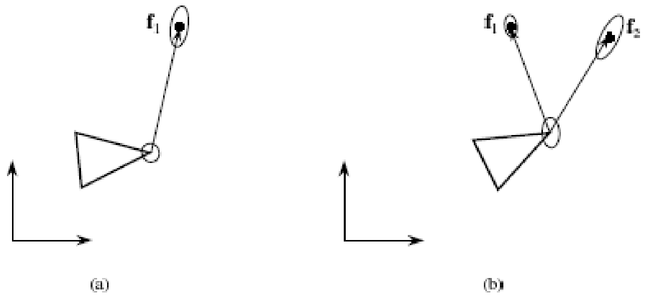
Basic SLAM Operation



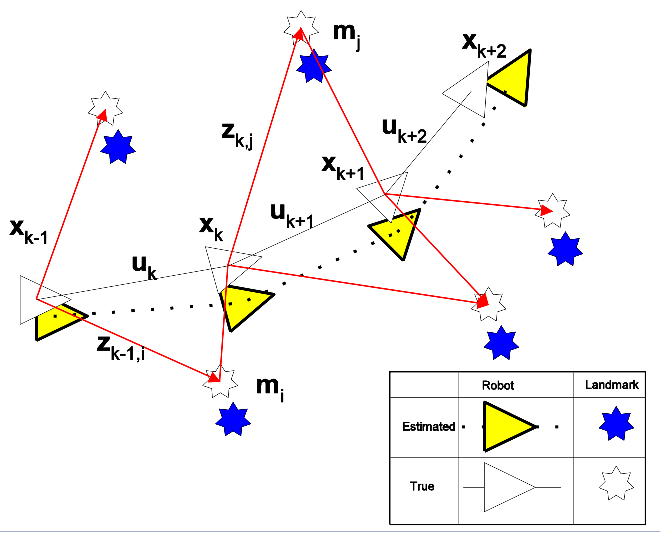
Basic SLAM Operation



Dependent Errors



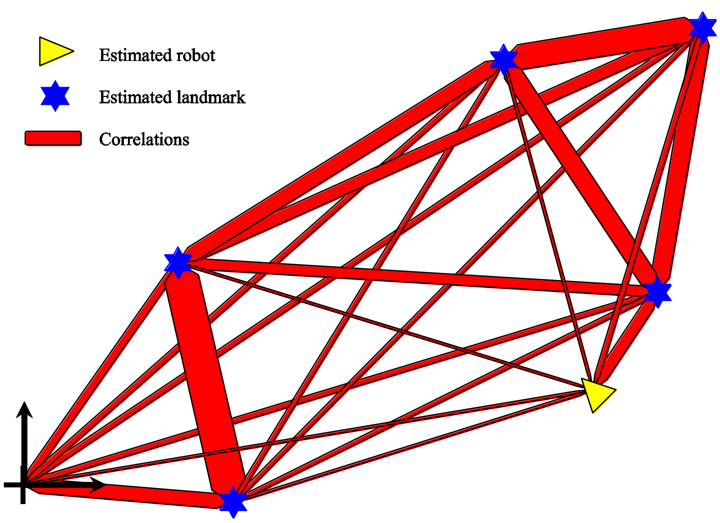
Correlated Estimates



SLAM Convergence

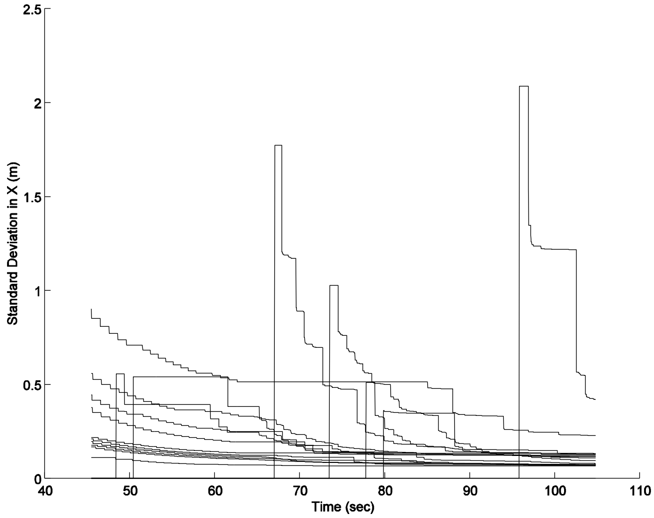
- An observation acts like a displacement to a spring system
  - Effect is greatest in a close neighbourhood
  - Effect on other landmarks diminishes with distance
  - Propagation depends on local stiffness (correlation) properties
- With each new observation the springs become increasingly (and monotonically) stiffer.
- In the limit, a rigid map of landmarks is obtained.
  - A perfect *relative* map of the environment
- The location accuracy of the robot is bounded by
  - The current quality of the map
  - The relative sensor measurement

Spring Analogy



Monotonic Convergence

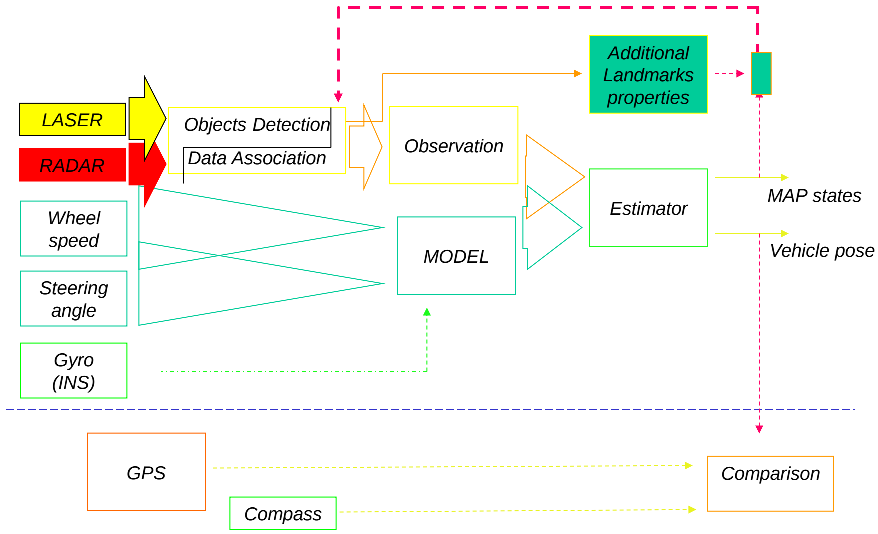
- With each new observation, the determinant decreases over the map and for any submatrix in the map.



Models

- Models are central to creating a representation of the world.
- Must have a mapping between sensed data (eg, laser, cameras, odometry) and the states of interest (eg, vehicle pose, stationary landmarks)
- Two essential model types:
  - Vehicle motion
  - Sensing of external objects

An Example System



States, Controls, Observations

Joint state with momentary pose

$$\mathbf{x}_k = \begin{bmatrix} \mathbf{x}_{v_k} \\ \mathbf{m}_1 \\ \vdots \\ \mathbf{m}_N \end{bmatrix}$$

Joint state with pose history

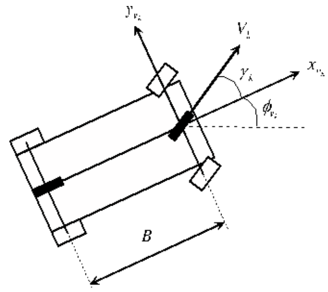
$$\mathbf{x}_k = \begin{bmatrix} \mathbf{x}_{v_k} \\ \mathbf{x}_{v_{k-1}} \\ \vdots \\ \mathbf{x}_{v_0} \\ \mathbf{m}_1 \\ \vdots \\ \mathbf{m}_N \end{bmatrix}$$

Control inputs: $\mathbf{U}_{0:k} = \{\mathbf{u}_1, \mathbf{u}_2, \cdots, \mathbf{u}_k\} = \{\mathbf{U}_{0:k-1}, \mathbf{u}_k\}$

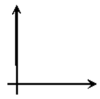
Observations: $\mathbf{Z}_{0:k} = \{\mathbf{z}_1, \mathbf{z}_2, \cdots, \mathbf{z}_k\} = \{\mathbf{Z}_{0:k-1}, \mathbf{z}_k\}$

Vehicle Motion Model

- Ackerman steered vehicles: Bicycle model



- Discrete time model:



$$\mathbf{x}_{v_k} = \mathbf{f}_v\left(\mathbf{x}_{v_{k-1}}, \mathbf{u}_k\right) = \begin{bmatrix} x_{v_{k-1}} + V_k \Delta T \cos(\phi_{v_{k-1}} + \gamma_k) \\ y_{v_{k-1}} + V_k \Delta T \sin(\phi_{v_{k-1}} + \gamma_k) \\ \phi_{v_{k-1}} + \frac{V_k \Delta T}{B} \sin(\gamma_k) \end{bmatrix}$$

SLAM Motion Model

$$\mathbf{x}_{v_k} = \mathbf{f}_v\left(\mathbf{x}_{v_{k-1}}, \mathbf{u}_k\right) = \begin{bmatrix} x_{v_{k-1}} + V_k \Delta T \cos(\phi_{v_{k-1}} + \gamma_k) \\ y_{v_{k-1}} + V_k \Delta T \sin(\phi_{v_{k-1}} + \gamma_k) \\ \phi_{v_{k-1}} + \frac{V_k \Delta T}{B} \sin(\gamma_k) \end{bmatrix}$$

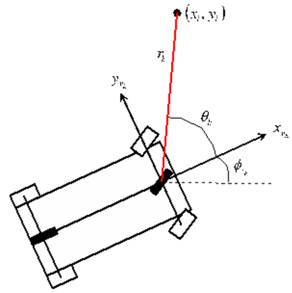
- Joint state: Landmarks are assumed stationary

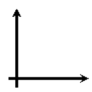
$$\mathbf{x}_k = \begin{bmatrix} \mathbf{f}_v(\mathbf{x}_{v_{k-1}}, \mathbf{u}_k) \\ \mathbf{m}_1 \\ \vdots \\ \mathbf{m}_N \end{bmatrix} \qquad \mathbf{x}_k = \begin{bmatrix} \mathbf{f}_v(\mathbf{x}_{v_{k-1}}, \mathbf{u}_k) \\ \mathbf{x}_{v_{k-1}} \\ \vdots \\ \mathbf{x}_{v_0} \\ \mathbf{m}_1 \\ \vdots \\ \mathbf{m}_N \end{bmatrix}$$

Observation Model

- Range-bearing measurement

$$\mathbf{z}_{i_k} = \mathbf{h}_i(\mathbf{x}_k) = \begin{bmatrix} \sqrt{(x_i - x_{v_k})^2 + (y_i - y_{v_k})^2} \\ \arctan \frac{y_i - y_{v_k}}{x_i - x_{v_k}} - \phi_{v_k} \end{bmatrix}$$





Applying Bayes to SLAM: Available Information

- States $\mathbf{x}_k$ (Hidden or inferred values)
  - Vehicle poses
  - Map; typically composed of discrete parts called landmarks or features
- Controls $\mathbf{U}_{0:k}$
  - Velocity
  - Steering angle
- Observations $\mathbf{Z}_{0:k}$
  - Range-bearing measurements

Augmentation: Adding new poses and landmarks

- Add new pose

$$\mathbf{x}_k = \begin{bmatrix} \mathbf{f}_v(\mathbf{x}_{v_{k-1}}, \mathbf{u}_k) \\ \mathbf{x}_{v_{k-1}} \\ \vdots \\ \mathbf{x}_{v_0} \\ \mathbf{m}_1 \\ \vdots \\ \mathbf{m}_N \end{bmatrix}$$

- Conditional probability is a Markov Model

$$\begin{aligned} p\left(\mathbf{x}_{v_k} | \mathbf{x}_{k-1}\right) &= \int p\left(\mathbf{x}_{v_k} | \mathbf{x}_{k-1}, \mathbf{u}_k\right) p\left(\mathbf{u}_k\right) d\mathbf{u}_k \\ &= \int \delta\left(\mathbf{x}_{v_k} - \mathbf{f}_v\left(\mathbf{x}_{v_{k-1}}, \mathbf{u}_k\right)\right) p\left(\mathbf{u}_k\right) d\mathbf{u}_k \\ &= p\left(\mathbf{x}_{v_k} | \mathbf{x}_{v_{k-1}}\right) \end{aligned}$$

Augmentation

$$\begin{aligned} p\left(\mathbf{x}_{v_k} | \mathbf{x}_{k-1}\right) &= \int p\left(\mathbf{x}_{v_k} | \mathbf{x}_{k-1}, \mathbf{u}_k\right) p\left(\mathbf{u}_k\right) d\mathbf{u}_k \\ &= \int \delta\left(\mathbf{x}_{v_k} - \mathbf{f}_v\left(\mathbf{x}_{v_{k-1}}, \mathbf{u}_k\right)\right) p\left(\mathbf{u}_k\right) d\mathbf{u}_k \\ &= p\left(\mathbf{x}_{v_k} | \mathbf{x}_{v_{k-1}}\right) \end{aligned}$$

- Product rule to create joint PDF $p(x_k)$

$$p\left(\mathbf{x}_{v_k}, \mathbf{x}_{k-1}\right) = p\left(\mathbf{x}_{v_k} | \mathbf{x}_{v_{k-1}}\right) p\left(\mathbf{x}_{v_{k-1}}, \ldots, \mathbf{x}_{v0}, \mathbf{m}_1, \ldots, \mathbf{m}_N\right)$$

- Same method applies to adding new landmark states

Marginalisation: Removing past poses and obsolete landmarks

- Augmenting with new pose and marginalising the old pose gives the classical SLAM prediction step

$$p\left(\mathbf{x}_{v_k}, \mathbf{m}_1, \ldots, \mathbf{m}_N\right) = \int p\left(\mathbf{x}_{v_k}, \mathbf{x}_{v_{k-1}}, \mathbf{m}_1, \ldots, \mathbf{m}_N\right) d\mathbf{x}_{v_{k-1}}$$

Fusion: Incorporating observation information

- Conditional PDF according to observation model

$$\begin{aligned} p\left(\mathbf{z}_{i_k} | \mathbf{x}_k\right) &= \int p\left(\mathbf{z}_{i_k} | \mathbf{x}_{v_k}, \mathbf{m}_i, \mathbf{r}_k\right) p\left(\mathbf{r}_k\right) d\mathbf{r}_k \\ &= \int \delta\left(\mathbf{z}_{i_k} - \mathbf{h}\left(\mathbf{x}_{v_k}, \mathbf{m}_i, \mathbf{r}_k\right)\right) p\left(\mathbf{r}_k\right) d\mathbf{r}_k \end{aligned}$$

- Bayes update:
  proportional to product of likelihood and prior

$$p\left(\mathbf{x}_k | \mathbf{Z}_{0:k}\right) = \frac{p\left(\mathbf{z}_{i_k} = \mathbf{z}_0 | \mathbf{x}_k\right) p\left(\mathbf{x}_k | \mathbf{Z}_{0:k-1}\right)}{p\left(\mathbf{z}_{i_k} = \mathbf{z}_0\right)}$$

Implementing Probabilistic SLAM

- The problem is that Bayesian operations are intractable in general.
  - General equations are good for analytical derivations, not good for implementation

- We need approximations
  - Linearised Gaussian systems (EKF, UKF, EIF, SAM)
  - Monte Carlo sampling methods (Rao-Blackwellised particle filters)

EKF SLAM

- The complicated Bayesian equations for augmentation, marginalisation, and fusion have simple and efficient closed form solutions for linear Gaussian systems

- For non-linear systems, just linearise
  - EKF, EIF: Jacobians
  - UKF: use deterministic samples

Kalman Implementation

- So can we just plug the process and observation models into the standard EKF equations and turn the crank?

- Several additional issues:
  - Structure of the SLAM problem permits more efficient implementation than naïve EKF.
  - Data association.
  - Feature initialisation.

Structure of SLAM

- Key property of stochastic SLAM
  - Largely a *parameter* estimation problem

- Since the map is stationary
  - No process model, no process noise

- For Gaussian SLAM
  - Uncertainty in each landmark reduces monotonically after landmark initialisation
  - Map converges

- Examine computational consequences of this structure in next session.

Data Association

- Before the Update Stage we need to determine if the feature we are observing is:
  - An old feature
  - A new feature

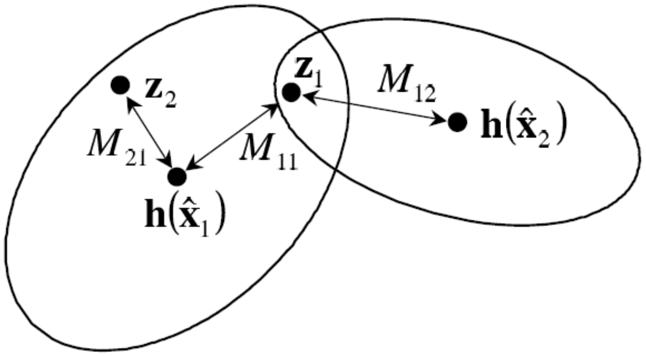
- If there is a match with only one known feature, the Update stage is run with this feature information.

$$\mu(k) = z(k) - h(\hat{x}(k/k-1))$$

$$S(k) = \nabla h_x(k) P(k/k-1) \nabla h_x^T(k) + R$$

$$\alpha = \mu^T(k) S^{-1}(k)\ \mu(k) < \chi^2_{0.95}$$

Validation Gating



New Features

- If there is no match then a potential new feature has been detected

- We do not want to incorporate a spurious observation as a new feature
  - It will not be observed again and will consume computational time and memory
  - It will add clutter, increasing risk of future mis-associations
  - The features are assumed to be static. We don't not want to accept dynamic objects as features: cars, people etc.

Acceptance of New Features

- **APPROACH 1**

- Get the feature in a list of potential features
- Incorporate the feature once it has been observed for a number of times

- Advantages:
  - Simple to implement
  - Appropriate for High Frequency external sensor

- Disadvantages:
  - Loss of information
  - Potentially a problem with sensor with small field of view: a feature may only be seen very few times

Acceptance of New Features

- **APPROACH 2**
  - The state vector is extended with past vehicle positions and the estimation of the cross-correlation between current and previous vehicle states is maintained. With this approach improved data association is possible by combining data form various points
    - J. J. Leonard and R. J. Rikoski. Incorporation of delayed decision making into stochastic mapping
    - Stephan Williams, PhD Thesis, 2001, University of Sydney
  - Advantages:
    - No Loss of Information
    - Well suited to low frequency external sensors (ratio between vehicle velocity and feature rate information)
    - Absolutely necessary for some sensor modalities (eg, range-only, bearing-only)
  - Disadvantages:
    - Cost of augmenting state with past poses
    - The implementation is more complicated

Incorporation of New Features

- **We have the vehicle states and previous map**

$$P_0 = \begin{bmatrix} P^0_{v,v} & P^0_{v,m} \\ P^0_{m,v} & P^0_{m,m} \end{bmatrix}$$

We observed a new feature and the covariance and cross-covariance terms need to be evaluated

$$P_1 = \begin{bmatrix} P^0_{v,v} & P^0_{v,m} & ? \\ P^0_{m,v} & P^0_{m,m} & ? \\ ? & ? & ? \end{bmatrix}$$

Incorporation of New Features

- Approach 1

$$P_0 = \begin{bmatrix} P_{vv}^0 & P_{vm}^0 & 0 \\ P_{mv}^0 & P_{mm}^0 & 0 \\ 0 & 0 & A \end{bmatrix}$$

With A very large

$$W(k) = P(k/k-1)\nabla h_x^T(k)S^{-1}(k)$$
$$S(k) = \nabla h_x(k)P(k/k-1)\nabla h_x^T(k) + R$$
$$P(k/k) = P(k/k-1) - W(k)S(k)W^T(k)$$

$$P_1 = \begin{bmatrix} P_{vv}^1 & P_{vm}^1 & P_{vn}^1 \\ P_{mv}^1 & P_{mm}^1 & P_{mn}^1 \\ P_{nv}^1 & P_{nm}^1 & P_{nn}^1 \end{bmatrix}$$

- **Easy to understand and implement**
- **Very large values of A may introduce numerical problems**

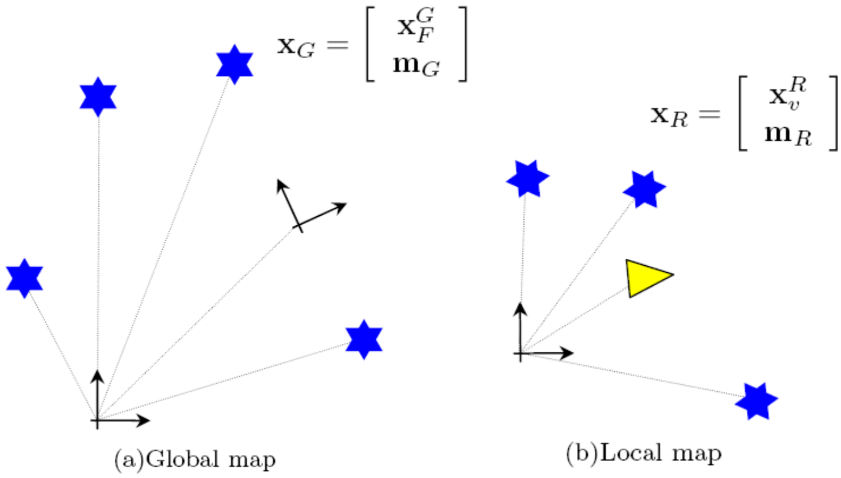
Analytical Approach

$$P_0 = \begin{bmatrix} P_{v,v}^0 & P_{v,m}^0 \\ P_{m,v}^0 & P_{m,m}^0 \end{bmatrix}$$

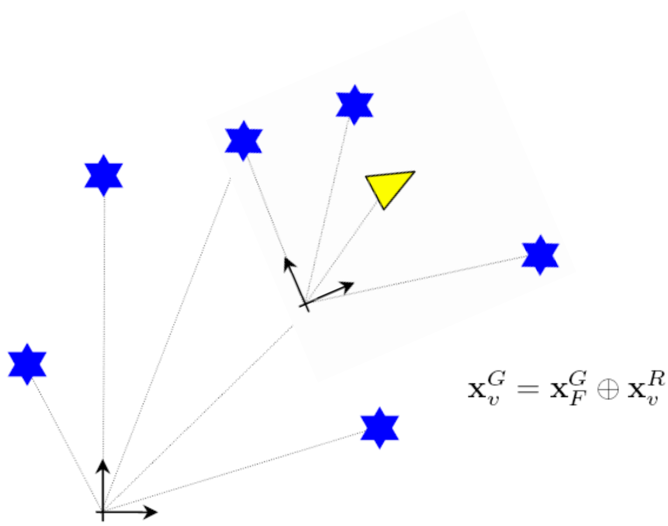
- **We can also evaluate the analytical expressions of the new terms**

$$P_1 = \begin{bmatrix} P_{v,v}^0 & P_{v,m}^0 & ? \\ P_{m,v}^0 & P_{m,m}^0 & ? \\ ? & ? & ? \end{bmatrix}$$

Constrained Local Submap Filter



CLSF Registration



CLSF Global Estimate