Robot Sensing: Feature Detection
(as Linear Observers)

METR 4202: Advanced Control & **Robotics**

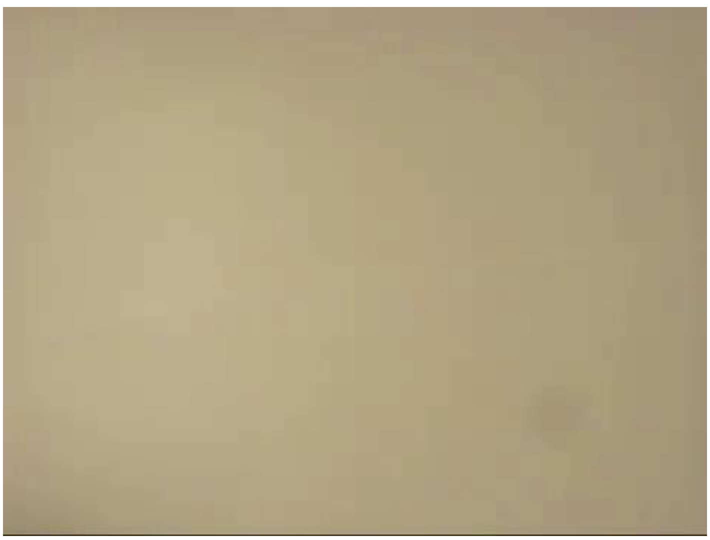
Dr Surya Singh -- Lecture # 7

September 9, 2015 – 9/9!

metr4202@itee.uq.edu.au

http://robotics.itee.uq.edu.au/~metr4202/

Cool Robotics Share



D. Wedge, *The Fundamental Matrix Song*

Schedule

Week	Date	Lecture (W: 12:05-1:50, 50-N201)
1	29-Jul	Introduction
2	5-Aug	Representing Position & Orientation & State (Frames, Transformation Matrices & Affine Transformations)
3	12-Aug	Robot Kinematics Review (& *Ekka Day*)
4	19-Aug	Robot Dynamics
5	26-Aug	Robot Sensing: Perception
6	2-Sep	Robot Sensing: Multiple View Geometry
7	**9-Sep**	**Robot Sensing: Feature Detection (as Linear Observers)**
8	16-Sep	Probabalistic Robotics: Localization
9	23-Sep	**Quiz** & Guest Lecture (SLAM?)
	30-Sep	*Study break*
10	7-Oct	Motion Planning
11	14-Oct	State-Space Modelling
12	21-Oct	Shaping the Dynamic Response
13	28-Oct	LQR + Course Review

Camera matrix calibration

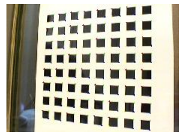
- Advantages:
  - very simple to formulate and solve
  - can recover K [R | t] from M using QR decomposition [Golub & VanLoan 96]
- Disadvantages:
  - doesn't compute internal parameters
  - more unknowns than true degrees of freedom
  - need a separate camera matrix for each new view

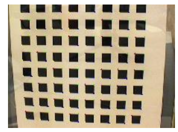
From Szeliski, *Computer Vision: Algorithms and Applications*

Multi-plane calibration

- Use several images of planar target held at unknown orientations [Zhang 99]
  - Compute plane homographies

$$\begin{bmatrix} u_i \\ v_i \\ 1 \end{bmatrix} \sim \mathbf{K} \begin{bmatrix} \mathbf{r}_1 & \mathbf{r}_2 & \mathbf{t} \end{bmatrix} \begin{bmatrix} x_i \\ y_i \\ 1 \end{bmatrix} \sim \mathbf{HX}$$



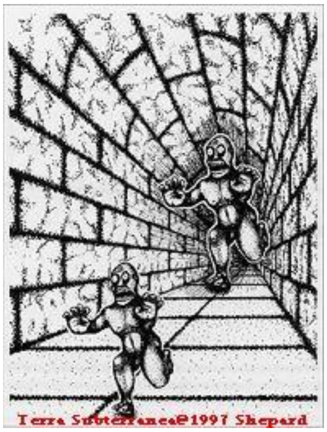


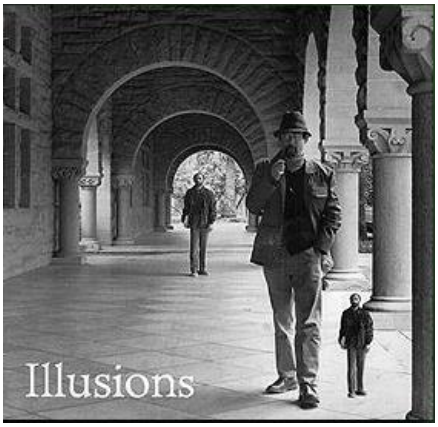
  - Solve for K-TK-1 from Hk's
    - 1plane if only f unknown
    - 2 planes if (f,uc,vc) unknown
    - 3+ planes for full K
  - Code available from Zhang and OpenCV

From Szeliski, *Computer Vision: Algorithms and Applications*

"Fundamental" Multi-View Geometry

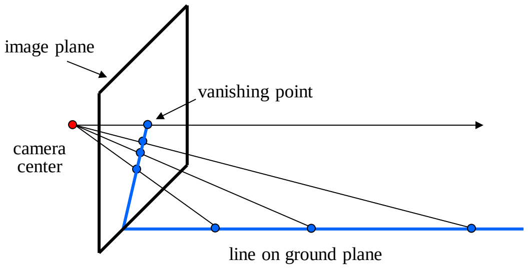
Fun With Vanishing Points



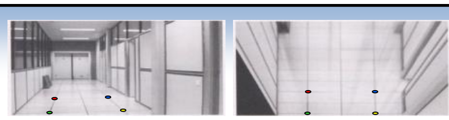


Slide from Szeliski, *Computer Vision: Algorithms and Applications*

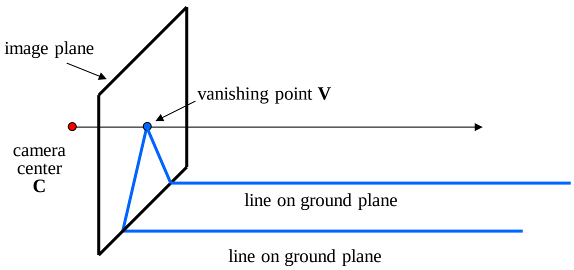
Vanishing Points (2D)



Vanishing Points

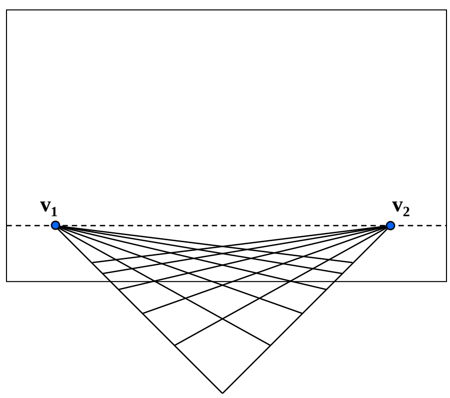


- Properties
  - Any two parallel lines have the same vanishing point
  - The ray from C through v point is parallel to the lines
  - An image may have more than one vanishing point



Vanishing Lines

- Multiple Vanishing Points
  - Any set of parallel lines on the plane define a vanishing point
  - The union of all of these vanishing points is the horizon line

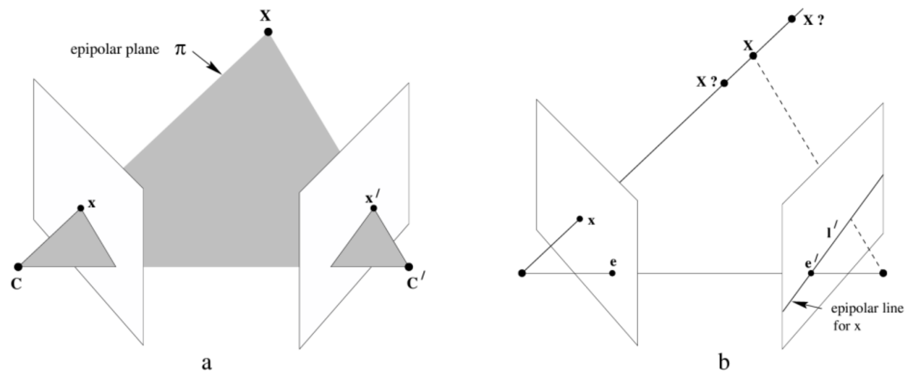


Stereo: epipolar geometry

- for two images (or images with collinear camera centers), can find epipolar lines
- epipolar lines are the projection of the pencil of planes passing through the centers
- Rectification: warping the input images (perspective transformation) so that epipolar lines are horizontal

Slide from Szeliski, *Computer Vision: Algorithms and Applications*

Two-View Geometry: Epipolar Plane



- **Epipole:** The *point* of intersection of the line joining the camera centres (the baseline) with the image plane. Equivalently, the epipole is the image in one view of the camera centre of the other view.
- **Epipolar plane** is a plane containing the baseline. There is a one-parameter family (a pencil) of epipolar planes
- **Epipolar line** is the intersection of an epipolar plane with the image plane. All epipolar lines intersect at the epipole. An epipolar plane intersects the left and right image planes in epipolar lines, and defines the correspondence between the lines.

Two-frame methods

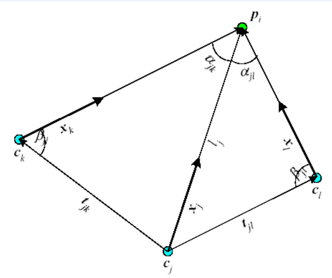
- Two main variants:
- Calibrated: "Essential matrix" E
  use ray directions (xi, xi')
- Uncalibrated: "Fundamental matrix" F

- [Hartley & Zisserman 2000]

From Szeliski, *Computer Vision: Algorithms and Applications*

Essential matrix

- Co-planarity constraint:
  - $x' \approx R\,x + t$
  - $[t]_\times x' \approx [t]_\times R\,x$
  - $x'\,[t]_\times x' \approx x'\,[t]_\times R\,x$
  - $x'\,E\,x = 0$ with $E = [t]_\times R$

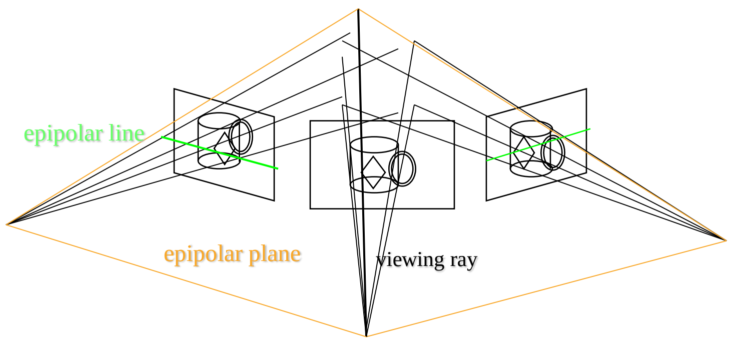


- Solve for **E** using least squares (SVD)
- t is the least singular vector of E
- R obtained from the other two s.v.s

From Szeliski, *Computer Vision: Algorithms and Applications*

Stereo: Epipolar geometry

- Match features along epipolar lines



Slide from Szeliski, *Computer Vision: Algorithms and Applications*

Fundamental Matrix

- The fundamental matrix is the algebraic representation of epipolar geometry.

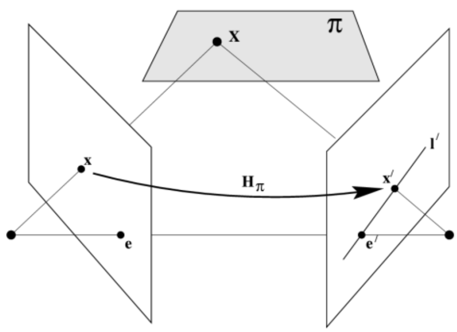


Fig. 9.5. *A point* $\mathbf{x}$ *in one image is transferred via the plane* π *to a matching point* $\mathbf{x}'$ *in the second image. The epipolar line through* $\mathbf{x}'$ *is obtained by joining* $\mathbf{x}'$ *to the epipole* $\mathbf{e}'$*. In symbols one may write* $\mathbf{x}' = H_\pi \mathbf{x}$ *and* $\mathbf{l}' = [\mathbf{e}']_\times \mathbf{x}' = [\mathbf{e}']_\times H_\pi \mathbf{x} = F\mathbf{x}$ *where* $F = [\mathbf{e}']_\times H_\pi$ *is the fundamental matrix.*

Fundamental matrix

- Camera calibrations are unknown
- x' F x = 0 with F = [e]× H = K'[t]× R K-1
- Solve for F using least squares (SVD)
  - re-scale (xi, xi') so that |xi|≈1/2 [Hartley]
- e (epipole) is still the least singular vector of F
- H obtained from the other two s.v.s
- "plane + parallax" (projective) reconstruction
- use self-calibration to determine K [Pollefeys]

From Szeliski, *Computer Vision: Algorithms and Applications*

Fundamental Matrix Example

- Suppose the camera matrices are those of a calibrated stereo rig with the world origin at the first camera

$$\mathtt{P} = \mathtt{K}[\mathtt{I} \mid \mathbf{0}] \qquad \mathtt{P}' = \mathtt{K}'[\mathtt{R} \mid \mathbf{t}].$$

- Then:

$$\mathtt{P}^{+} = \begin{bmatrix} \mathtt{K}^{-1} \\ \mathbf{0}^{\mathsf{T}} \end{bmatrix} \qquad \mathbf{C} = \begin{pmatrix} \mathbf{0} \\ 1 \end{pmatrix}$$

- Epipoles are at:

$$\mathbf{e} = \mathtt{P}\begin{pmatrix} -\mathtt{R}^{\mathsf{T}}\mathbf{t} \\ 1 \end{pmatrix} = \mathtt{K}\mathtt{R}^{\mathsf{T}}\mathbf{t} \qquad \mathbf{e}' = \mathtt{P}'\begin{pmatrix} \mathbf{0} \\ 1 \end{pmatrix} = \mathtt{K}'\mathbf{t}.$$

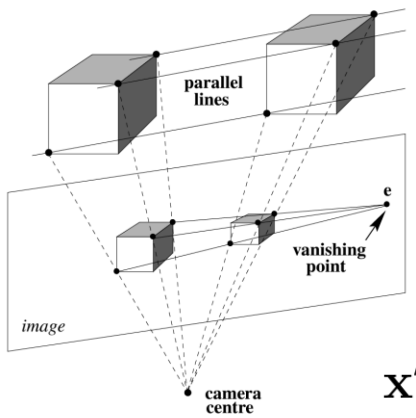
∴

$$\mathtt{F} = [\mathbf{e}']_{\times}\mathtt{K}'\mathtt{R}\mathtt{K}^{-1} = \mathtt{K}'^{-\mathsf{T}}[\mathbf{t}]_{\times}\mathtt{R}\mathtt{K}^{-1} = \mathtt{K}'^{-\mathsf{T}}\mathtt{R}[\mathtt{R}^{\mathsf{T}}\mathbf{t}]_{\times}\mathtt{K}^{-1} = \mathtt{K}'^{-\mathsf{T}}\mathtt{R}\mathtt{K}^{\mathsf{T}}[\mathbf{e}]_{\times}$$

Summary of fundamental matrix properties

- F is a rank 2 homogeneous matrix with 7 degrees of freedom.
- **Point correspondence**: If $\mathbf{x}$ and $\mathbf{x}'$ are corresponding image points, then $\mathbf{x}'^{\mathsf{T}}\mathtt{F}\mathbf{x} = 0$.
- **Epipolar lines**:
  - ◇ $\mathbf{l}' = \mathtt{F}\mathbf{x}$ is the epipolar line corresponding to $\mathbf{x}$.
  - ◇ $\mathbf{l} = \mathtt{F}^{\mathsf{T}}\mathbf{x}'$ is the epipolar line corresponding to $\mathbf{x}'$.
- **Epipoles**:
  - ◇ $\mathtt{F}\mathbf{e} = \mathbf{0}$.
  - ◇ $\mathtt{F}^{\mathsf{T}}\mathbf{e}' = \mathbf{0}$.
- **Computation from camera matrices** $\mathtt{P}, \mathtt{P}'$:
  - ◇ General cameras,
    $\mathtt{F} = [\mathbf{e}']_\times \mathtt{P}'\mathtt{P}^+$, where $\mathtt{P}^+$ is the pseudo-inverse of $\mathtt{P}$, and $\mathbf{e}' = \mathtt{P}'\mathbf{C}$, with $\mathtt{P}\mathbf{C} = \mathbf{0}$.
  - ◇ Canonical cameras, $\mathtt{P} = [\mathtt{I} \mid \mathbf{0}]$, $\mathtt{P}' = [\mathtt{M} \mid \mathbf{m}]$,
    $\mathtt{F} = [\mathbf{e}']_\times \mathtt{M} = \mathtt{M}^{-\mathsf{T}}[\mathbf{e}]_\times$, where $\mathbf{e}' = \mathbf{m}$ and $\mathbf{e} = \mathtt{M}^{-1}\mathbf{m}$.
  - ◇ Cameras not at infinity $\mathtt{P} = \mathtt{K}[\mathtt{I} \mid \mathbf{0}]$, $\mathtt{P}' = \mathtt{K}'[\mathtt{R} \mid \mathbf{t}]$,
    $\mathtt{F} = \mathtt{K}'^{-\mathsf{T}}[\mathbf{t}]_\times \mathtt{R}\mathtt{K}^{-1} = [\mathtt{K}'\mathbf{t}]_\times \mathtt{K}'\mathtt{R}\mathtt{K}^{-1} = \mathtt{K}'^{-\mathsf{T}}\mathtt{R}\mathtt{K}^{\mathsf{T}}[\mathtt{K}\mathtt{R}^{\mathsf{T}}\mathbf{t}]_\times$.

Fundamental Matrix & Motion



$$\mathbf{x}'^{\mathsf{T}}\mathtt{F}\mathbf{x} = 0.$$

- Under a pure translational camera motion, 3D points appear to slide along parallel rails. The images of these parallel lines intersect in a vanishing point corresponding to the translation direction. The epipole **e** is the vanishing point.

Finding correspondences

- Apply feature matching criterion (e.g., correlation or Lucas-Kanade) at all pixels simultaneously
- Search only over epipolar lines (many fewer candidate positions)

Slide from Szeliski, *Computer Vision: Algorithms and Applications*

Matching criteria

- Raw pixel values (correlation)
- Band-pass filtered images [Jones & Malik 92]
- "Corner" like features [Zhang, …]
- Edges [many people…]
- Gradients [Seitz 89; Scharstein 94]
- Rank statistics [Zabih & Woodfill 94]

Slide from Szeliski, *Computer Vision: Algorithms and Applications*

Feature Detection

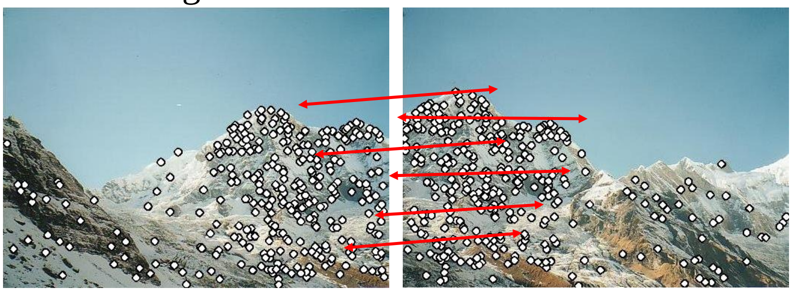
“A Rose By Any Other Name?

3 8 1 7 6 7 4 7 8 3 5 9 5 3 6 3 7 4 4 6 9 3 8 7 9 0 3 6 3 2 6 6 5 6 0 3 4 2 6 8 3 8 1…
7 6 7 4 7 8 3 5 9 5 3 6 3 7 4 4 6 9 3 8 7 9 0 3 6 3 2 6 6 5 6 0 3 4 2 6 8

– SIFT

Why extract features?

- **Object detection**
- Robot Navigation
- Scene Recognition



- Steps:
  - Extract Features
  - Match Features

Adopted drom S. Lazebnik, Gang Hua (CS 558)

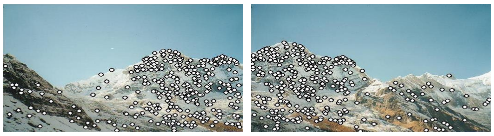
Why extract features? [2]

- Panorama stitching…
  - → Step 3: Align images

Adopted from S. Lazebnik, Gang Hua (CS 558)

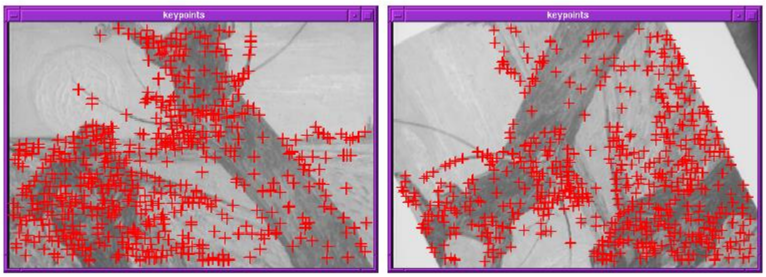
Characteristics of good features

- Repeatability
  - The same feature can be found in several images despite geometric and photometric transformations
- Saliency
  - Each feature is distinctive
- Compactness and efficiency
  - Many fewer features than image pixels
- Locality
  - A feature occupies a relatively small area of the image; robust to clutter and occlusion



Adopted from S. Lazebnik, Gang Hua (CS 558)

Finding Corners



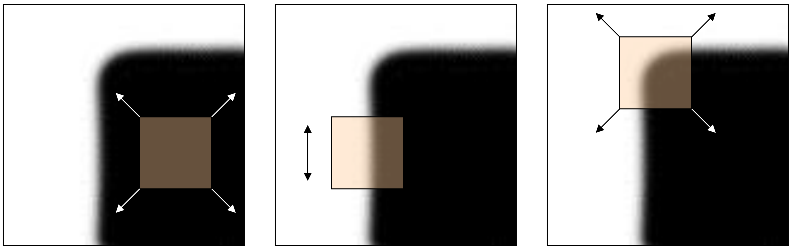
- Key property: in the region around a corner, image gradient has two or more dominant directions
- Corners are repeatable and distinctive

C.Harris and M.Stephens. "A Combined Corner and Edge Detector.“ *Proceedings of the 4th Alvey Vision Conference*: pages 147—151, 1988.

Adopted from S. Lazebnik, Gang Hua (CS 558)

Corner Detection: Basic Idea

- Look through a window
- Shifting a window in any direction should give a large change in intensity



"flat" region:
no change in
all directions

"edge":
no change along
the edge direction

"corner":
significant change
in all directions

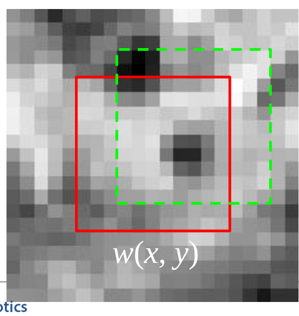
Source: A. Efros

Corner Detection: Mathematics

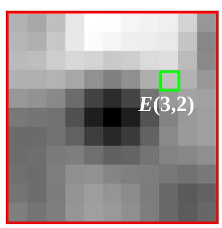
Change in appearance of window $w(x,y)$
for the shift $[u,v]$:

$$E(u,v) = \sum_{x,y} w(x,y)\left[I(x+u,y+v) - I(x,y)\right]^2$$

$I(x, y)$



$E(u, v)$



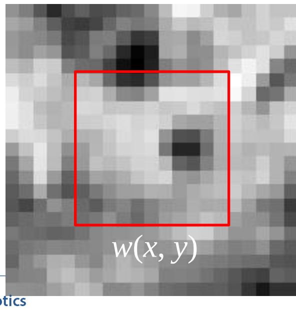
Adopted from
S. Lazebnik,
Gang Hua (CS 558)

Corner Detection: Mathematics

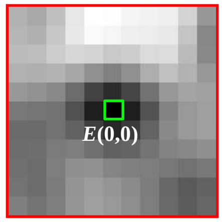
Change in appearance of window $w(x,y)$
for the shift $[u,v]$:

$$E(u,v) = \sum_{x,y} w(x,y)\left[I(x+u, y+v) - I(x,y)\right]^2$$

$I(x, y)$



$E(u, v)$



Adopted from
S. Lazebnik,
Gang Hua (CS 558)

Corner Detection: Mathematics

Change in appearance of window $w(x,y)$
for the shift $[u,v]$:

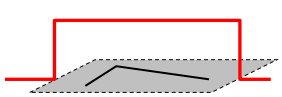
$$E(u,v) = \sum_{x,y} w(x,y)\left[I(x+u, y+v) - I(x,y)\right]^2$$

Window function

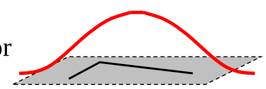
Shifted intensity

Intensity

Window function $w(x,y)$ =



or



1 in window, 0 outside

Gaussian

Adopted from
S. Lazebnik,
Gang Hua (CS 558)

Source: R. Szeliski

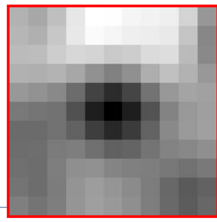
Corner Detection: Mathematics

Change in appearance of window $w(x,y)$
for the shift $[u,v]$:

$$E(u,v) = \sum_{x,y} w(x,y)\left[I(x+u, y+v) - I(x,y)\right]^2$$

We want to find out how this function behaves for small shifts

$E(u, v)$



Adopted from
S. Lazebnik,
Gang Hua ([CS 558](CS 558))

Corner Detection: Mathematics

Change in appearance of window $w(x,y)$
for the shift $[u,v]$:

$$E(u,v) = \sum_{x,y} w(x,y)\left[I(x+u, y+v) - I(x,y)\right]^2$$

We want to find out how this function behaves for small shifts

$$E(u,v) \approx E(0,0) + [u \;\; v]\begin{bmatrix} E_u(0,0) \\ E_v(0,0) \end{bmatrix} + \frac{1}{2}[u \;\; v]\begin{bmatrix} E_{uu}(0,0) & E_{uv}(0,0) \\ E_{uv}(0,0) & E_{vv}(0,0) \end{bmatrix}\begin{bmatrix} u \\ v \end{bmatrix}$$

Local quadratic approximation of $E(u,v)$ in the neighborhood of (0,0) is given by the *second-order Taylor expansion*:

Adopted from
S. Lazebnik,
Gang Hua ([CS 558](CS 558))

Corner Detection: Mathematics

$$E(u,v) = \sum_{x,y} w(x,y)\left[I(x+u, y+v) - I(x,y)\right]^2$$

Second-order Taylor expansion of $E(u,v)$ about (0,0):

$$E(u,v) \approx E(0,0) + [u \;\; v]\begin{bmatrix} E_u(0,0) \\ E_v(0,0) \end{bmatrix} + \frac{1}{2}[u \;\; v]\begin{bmatrix} E_{uu}(0,0) & E_{uv}(0,0) \\ E_{uv}(0,0) & E_{vv}(0,0) \end{bmatrix}\begin{bmatrix} u \\ v \end{bmatrix}$$

$$E_u(u,v) = \sum_{x,y} 2w(x,y)\left[I(x+u, y+v) - I(x,y)\right] I_x(x+u, y+v)$$

$$E_{uu}(u,v) = \sum_{x,y} 2w(x,y) I_x(x+u, y+v) I_x(x+u, y+v) + \sum_{x,y} 2w(x,y)\left[I(x+u, y+v) - I(x,y)\right] I_{xx}(x+u, y+v)$$

$$E_{uv}(u,v) = \sum_{x,y} 2w(x,y) I_y(x+u, y+v) I_x(x+u, y+v) + \sum_{x,y} 2w(x,y)\left[I(x+u, y+v) - I(x,y)\right] I_{xy}(x+u, y+v)$$

Adopted from
S. Lazebnik,
Gang Hua (CS 558)

Corner Detection: Mathematics

$$E(u,v) = \sum_{x,y} w(x,y)\left[I(x+u, y+v) - I(x,y)\right]^2$$

Second-order Taylor expansion of $E(u,v)$ about (0,0):

$$E(u,v) \approx [u \;\; v]\begin{bmatrix} \sum_{x,y} w(x,y) I_x^2(x,y) & \sum_{x,y} w(x,y) I_x(x,y) I_y(x,y) \\ \sum_{x,y} w(x,y) I_x(x,y) I_y(x,y) & \sum_{x,y} w(x,y) I_y^2(x,y) \end{bmatrix}\begin{bmatrix} u \\ v \end{bmatrix}$$

$$E(0,0) = 0$$

$$E_u(0,0) = 0$$

$$E_v(0,0) = 0$$

$$E_{uu}(0,0) = \sum_{x,y} 2w(x,y) I_x(x,y) I_x(x,y)$$

$$E_{vv}(0,0) = \sum_{x,y} 2w(x,y) I_y(x,y) I_y(x,y)$$

$$E_{uv}(0,0) = \sum_{x,y} 2w(x,y) I_x(x,y) I_y(x,y)$$

Adopted from
S. Lazebnik,
Gang Hua (CS 558)

Harris detector: Steps

- Compute Gaussian derivatives at each pixel
- Compute second moment matrix M in a Gaussian window around each pixel
- Compute corner response function R
- Threshold R
- Find local maxima of response function (nonmaximum suppression)

C.Harris and M.Stephens. "A Combined Corner and Edge Detector." *Proceedings of the 4th Alvey Vision Conference*: pages 147—151, 1988.

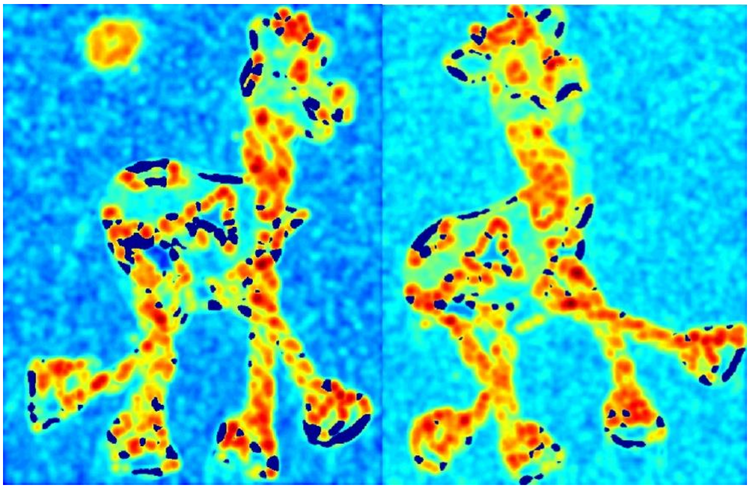
Adopted from
S. Lazebnik,
Gang Hua (CS 558)

Harris Detector: Steps

Adopted from S. Lazebnik, Gang Hua (CS 558)

Harris Detector: Steps

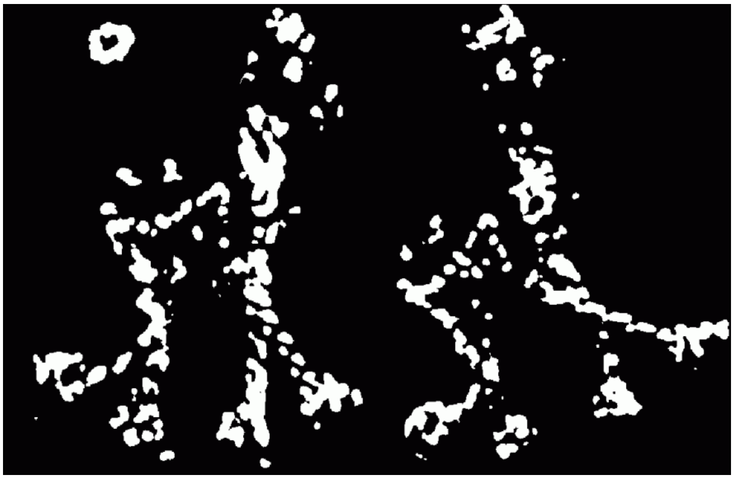
Compute corner response *R*



Adopted from S. Lazebnik, Gang Hua (CS 558)

Harris Detector: Steps

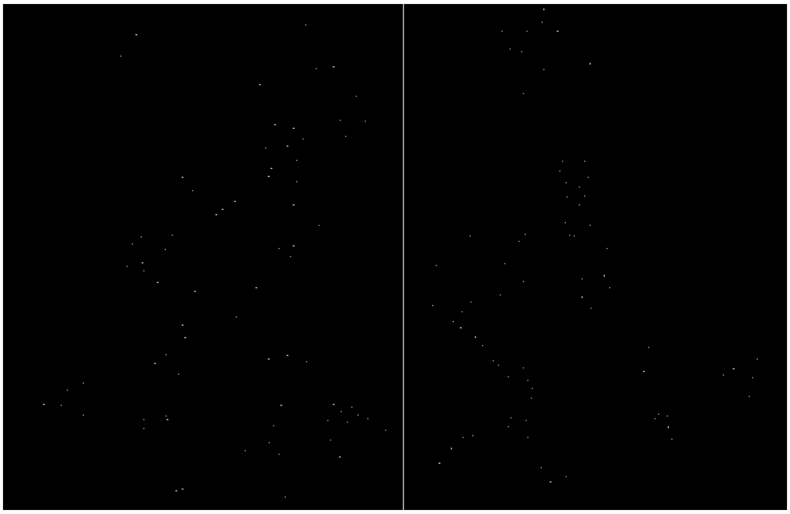
Find points with large corner response: *R*>threshold



Adopted from S. Lazebnik, Gang Hua (CS 558)

Harris Detector: Steps

Take only the points of local maxima of *R*



Adopted from S. Lazebnik, Gang Hua (CS 558)

Harris Detector: Steps

Adopted from S. Lazebnik, Gang Hua (CS 558)

Invariance and covariance

- We want corner locations to be invariant to photometric transformations and covariant to geometric transformations
  - Invariance: image is transformed and corner locations do not change
  - Covariance: if we have two transformed versions of the same image, features should be detected in corresponding locations

Adopted from S. Lazebnik, Gang Hua (CS 558)

RANdom SAmple Consensus

1. Repeatedly select a small (minimal) subset of correspondences
2. Estimate a solution (in this case a the line)
3. Count the number of "inliers", $|e|<\Theta$ (for LMS, estimate med($|e|$))
4. Pick the *best* subset of inliers
5. Find a complete least-squares solution

- Related to least median squares
- See also: MAPSAC (Maximum *A Posteriori* SAmple Consensus)

From Szeliski, *Computer Vision: Algorithms and Applications*

Scale Invariant Feature Transform

Basic idea:

- Take 16x16 square window around detected feature
- Compute edge orientation (angle of the gradient - 90°) for each pixel
- Throw out weak edges (threshold gradient magnitude)
- Create histogram of surviving edge orientations

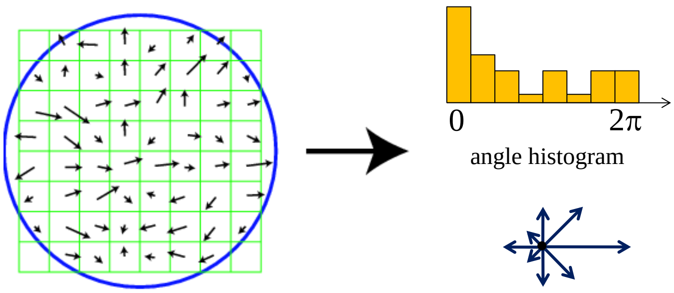


Image gradients

Adapted from slide by David Lowe

SIFT descriptor

Full version

- Divide the 16x16 window into a 4x4 grid of cells (2x2 case shown below)
- Compute an orientation histogram for each cell
- 16 cells * 8 orientations = 128 dimensional descriptor

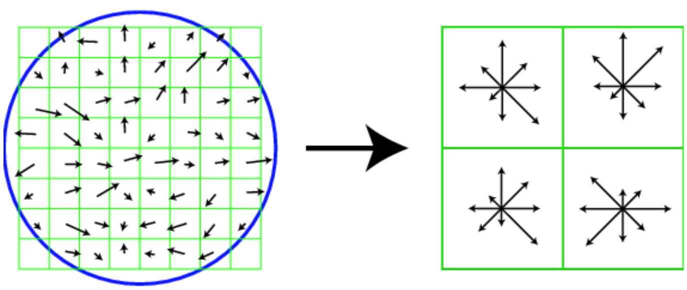


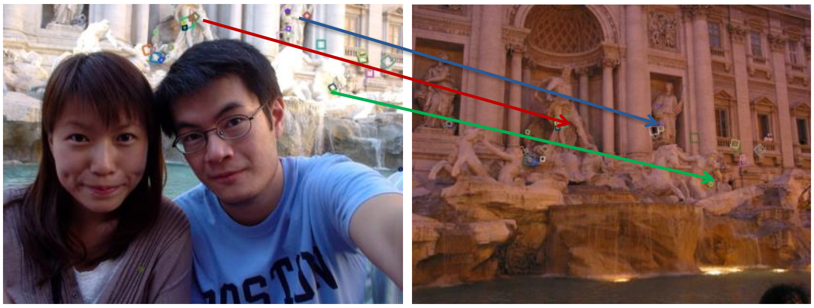
Image gradients

Keypoint descriptor

Adapted from slide by David Lowe

Properties of SIFT

- Extraordinarily robust matching technique
  - Can handle changes in viewpoint
    - Up to about 60 degree out of plane rotation
  - Can handle significant changes in illumination
    - Sometimes even day vs. night (below)
  - Fast and efficient—can run in real time
  - Lots of code available
    - http://people.csail.mit.edu/albert/ladypack/wiki/index.php/Known_implementations_of_SIFT



From David Lowe and Szeliski, *Computer Vision: Algorithms and Applications*

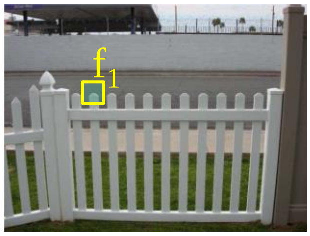
Feature matching

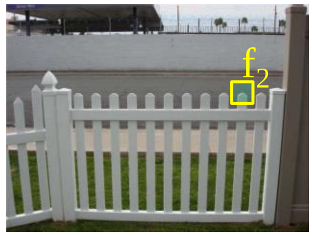
- Given a feature in I_1, how to find the best match in I_2?
  1. Define distance function that compares two descriptors
  2. Test all the features in I_2, find the one with min distance

From Szeliski, *Computer Vision: Algorithms and Applications*

Feature distance

- How to define the difference between two features f_1, f_2?
  - Simple approach is $SSD(f_1, f_2)$
    - sum of square differences between entries of the two descriptors
    - can give good scores to very ambiguous (bad) matches



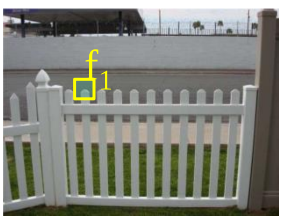


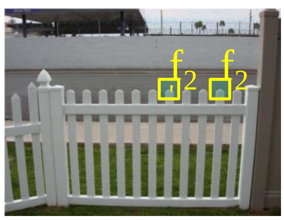
I_1 I_2

From Szeliski, *Computer Vision: Algorithms and Applications*

Feature distance

- How to define the difference between two features f_1, f_2?
  - Better approach: ratio distance = $SSD(f_1, f_2) / SSD(f_1, f_2')$
    - f_2 is best SSD match to f_1 in I_2
    - f_2' is 2nd best SSD match to f_1 in I_2
    - gives small values for ambiguous matches



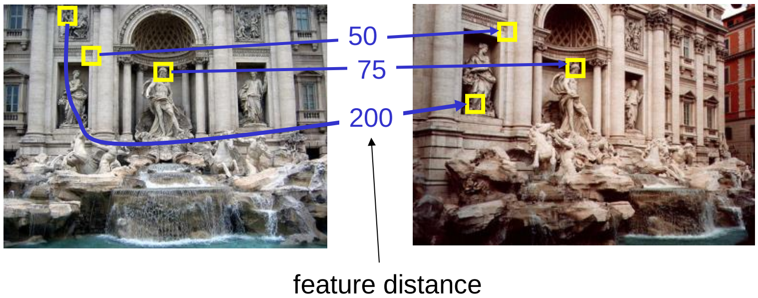


I_1 I_2

From Szeliski, *Computer Vision: Algorithms and Applications*

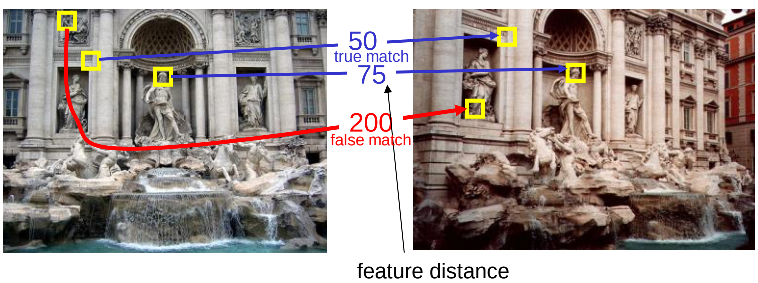
Evaluating the results

- How can we measure the performance of a feature matcher?



From Szeliski, *Computer Vision: Algorithms and Applications*

True/false positives



- The distance threshold affects performance
  - True positives = # of detected matches that are correct
    - Suppose we want to maximize these—how to choose threshold?
  - False positives = # of detected matches that are incorrect
    - Suppose we want to minimize these—how to choose threshold?

From Szeliski, *Computer Vision: Algorithms and Applications*

Levenberg-Marquardt

- Iterative non-linear least squares [Press'92]
  - Linearize measurement equations

$$\begin{aligned}\hat{u}_i &= f(\mathbf{m}, \mathbf{x}_i) + \frac{\partial f}{\partial \mathbf{m}} \Delta \mathbf{m} \\ \hat{v}_i &= g(\mathbf{m}, \mathbf{x}_i) + \frac{\partial g}{\partial \mathbf{m}} \Delta \mathbf{m}\end{aligned}$$

  - Substitute into log-likelihood equation: quadratic cost function in Dm

$$\sum_i \sigma_i^{-2} (\hat{u}_i - u_i + \frac{\partial f}{\partial \mathbf{m}} \Delta \mathbf{m})^2 + \cdots$$

From Szeliski, *Computer Vision: Algorithms and Applications*

Levenberg-Marquardt

- What if it doesn't converge?
  - Multiply diagonal by (1 + l), increase l until it does
  - Halve the step size Dm (my favorite)
  - Use line search
  - Other ideas?
- Uncertainty analysis: covariance S = A-1
- Is maximum likelihood the best idea?
- How to start in vicinity of global minimum?

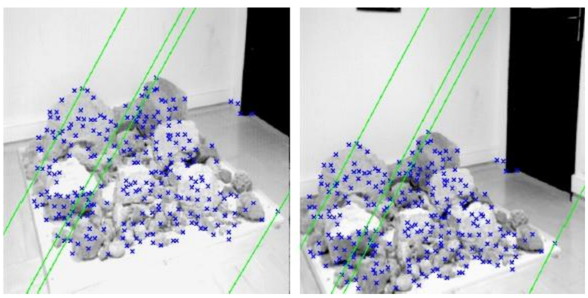
From Szeliski, *Computer Vision: Algorithms and Applications*

Feature Based Stereo

METR 4202: **Robotics** 9 September 201558

Feature-based stereo

- Match “corner” (interest) points



- Interpolate complete solution

Slide from Szeliski, *Computer Vision: Algorithms and Applications*

METR 4202: **Robotics** 9 September 201559

SFM: Structure from Motion
(& Cool Robotics Share (this week))

Structure [from] Motion

- Given a set of feature tracks,
  estimate the 3D structure and 3D (camera) motion.

- Assumption: orthographic projection

- Tracks: (u_{fp}, v_{fp}), f: frame, p: point
- Subtract out **mean** 2D position…

  $\mathbf{i}_f$: rotation, $\mathbf{s}_p$: position

$$u_{fp} = i_f^T s_p, v_{fp} = j_f^T s_p$$

From Szeliski, *Computer Vision: Algorithms and Applications*

Structure from motion

- How many points do we need to match?
- 2 frames:
  - (R,t): 5 dof + 3n point locations ≤
  - 4n point measurements ⇒
  - n ≥ 5
- k frames:
  - 6(k–1)-1 + 3n ≤ 2kn
- always want to use many more

$$\hat{u}_{ij} = f(\boxed{\mathrm{K}, \mathrm{R}_j, \mathrm{t}_j, \mathrm{x}_i})$$
$$\hat{v}_{ij} = g(\boxed{\mathrm{K}, \mathrm{R}_j, \mathrm{t}_j, \mathrm{x}_i})$$

From Szeliski, *Computer Vision: Algorithms and Applications*

Measurement equations

- Measurement equations

$$u_{fp} = \boldsymbol{i}_f^T \boldsymbol{s}_p \qquad \boldsymbol{i}_f\text{: rotation, } \boldsymbol{s}_p\text{: position}$$
$$v_{fp} = \boldsymbol{j}_f^T \boldsymbol{s}_p$$

- Stack them up…

$$\boldsymbol{W} = \boldsymbol{R}\,\boldsymbol{S}$$
$$\boldsymbol{R} = (\boldsymbol{i}_1,\ldots,\boldsymbol{i}_F, \boldsymbol{j}_1,\ldots,\boldsymbol{j}_F)^T$$
$$\boldsymbol{S} = (\boldsymbol{s}_1,\ldots,\boldsymbol{s}_P)$$

From Szeliski, *Computer Vision: Algorithms and Applications*

Factorization

$$W = R_{2F\times3}\, S_{3\times P}$$

SVD

$$W = U\Lambda V \quad \Lambda \text{ must be rank 3}$$

$$W' = (U\Lambda^{1/2})(\Lambda^{1/2} V) \quad = U'V'$$

Make R orthogonal

$$R = QU',\ \ S = Q^{-1}V'$$

$$i_f^T Q^T Q i_f = 1 \ \ldots$$

From Szeliski, [Computer Vision: Algorithms and Applications]()

Results

- Look at paper figures…

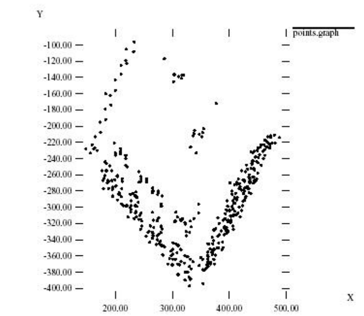


Figure 4.5: A view of the computed shape from approximately above the building (compare with figure 4.6).

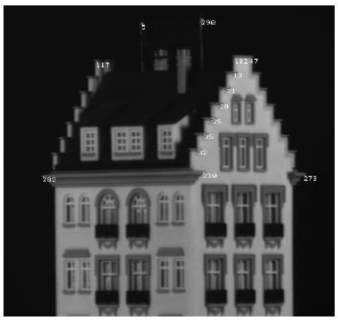


Figure 4.7: For a quantitative evaluation, distances between the features shown in the picture were measured on the actual model, and compared with the computed results. The comparison is shown in figure 4.8.

From Szeliski, [Computer Vision: Algorithms and Applications]()

Bundle Adjustment

- What makes this non-linear minimization hard?
  - many more parameters: potentially slow
  - poorer conditioning (high correlation)
  - potentially lots of outliers
  - gauge (coordinate) freedom

$$
\begin{aligned}
\hat{u}_{ij} &= f(\boxed{\mathbf{K}, \mathbf{R}_j, \mathbf{t}_j, \mathbf{x}_i}) \\
\hat{v}_{ij} &= g(\boxed{\mathbf{K}, \mathbf{R}_j, \mathbf{t}_j, \mathbf{x}_i})
\end{aligned}
$$

From Szeliski, *Computer Vision: Algorithms and Applications*

More Cool Robotics Share!