Motion Planning

METR 4202: Advanced Control & **Robotics**

Dr Surya Singh -- Lecture # 10

October 7, 2015

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Schedule

Week	Date	Lecture (W: 12:05-1:50, 50-N201)
1	29-Jul	Introduction
2	5-Aug	Representing Position & Orientation & State (Frames, Transformation Matrices & Affine Transformations)
3	12-Aug	Robot Kinematics Review (& *Ekka Day*)
4	19-Aug	Robot Dynamics
5	26-Aug	Robot Sensing: Perception
6	2-Sep	Robot Sensing: Multiple View Geometry
7	9-Sep	Robot Sensing: Feature Detection (as Linear Observers)
8	16-Sep	Probabilistic Robotics: Localization
9	23-Sep	Quiz
	30-Sep	*Study break*
10	**7-Oct**	**Motion Planning**
11	14-Oct	State-Space Modelling
12	21-Oct	Shaping the Dynamic Response
13	28-Oct	LQR + Course Review


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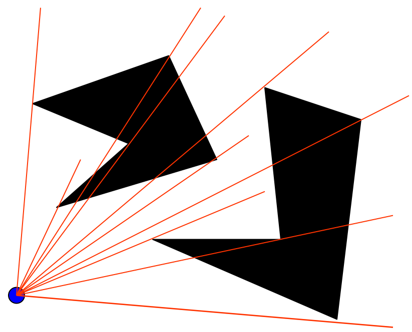
Now How Do We Use This To Get Somewhere?
Motion Planning

Path-Planning Approaches

- Roadmap
  Represent the connectivity of the free space by a network of 1-D curves
- Cell decomposition
  Decompose the free space into simple cells and represent the connectivity of the free space by the adjacency graph of these cells
- Potential field
  Define a function over the free space that has a global minimum at the goal configuration and follow its steepest descent

Slide from Latombe, CS326A

I. Rotational Sweep



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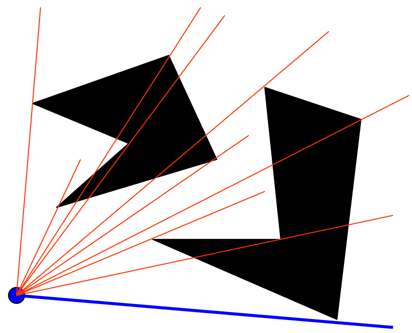
Rotational Sweep



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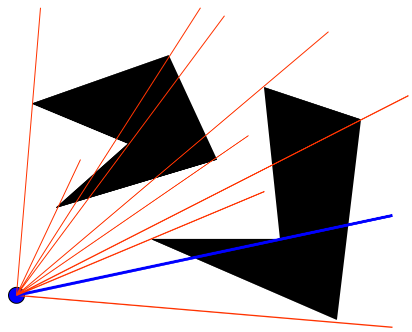

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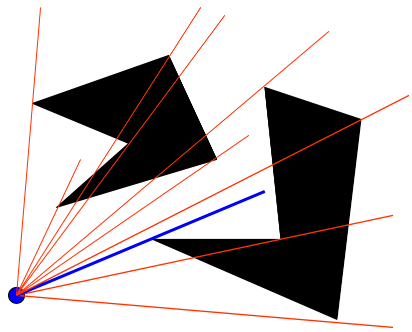

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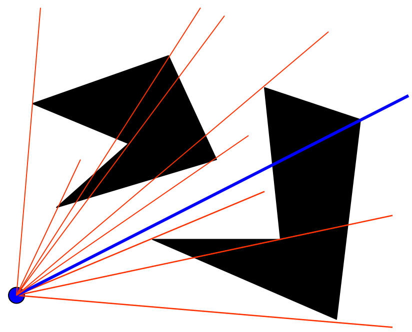

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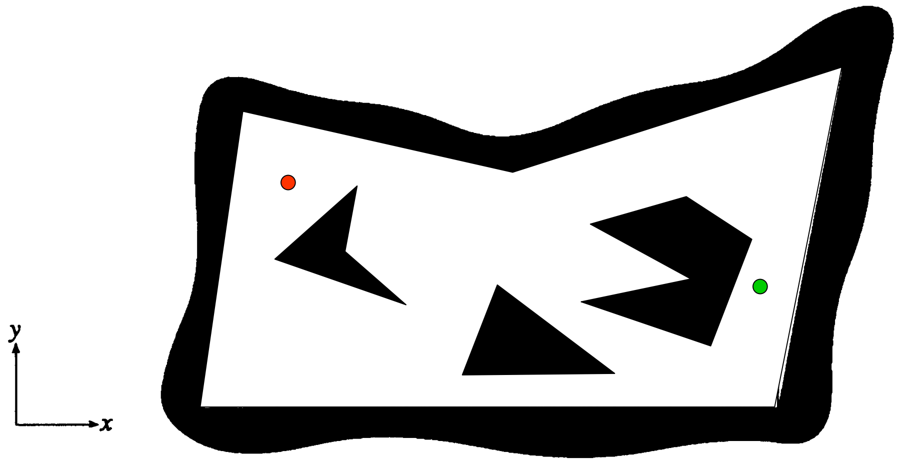
II. Cell-Decomposition Methods

Two classes of methods:

- Exact cell decomposition
  - The free space **F** is represented by a collection of non-overlapping cells whose union is exactly **F**
  - Example: trapezoidal decomposition
- Approximate cell decomposition
  - **F** is represented by a collection of non-overlapping cells whose union is contained in **F** Examples: quadtree, octree, 2n-tree

Trapezoidal decomposition



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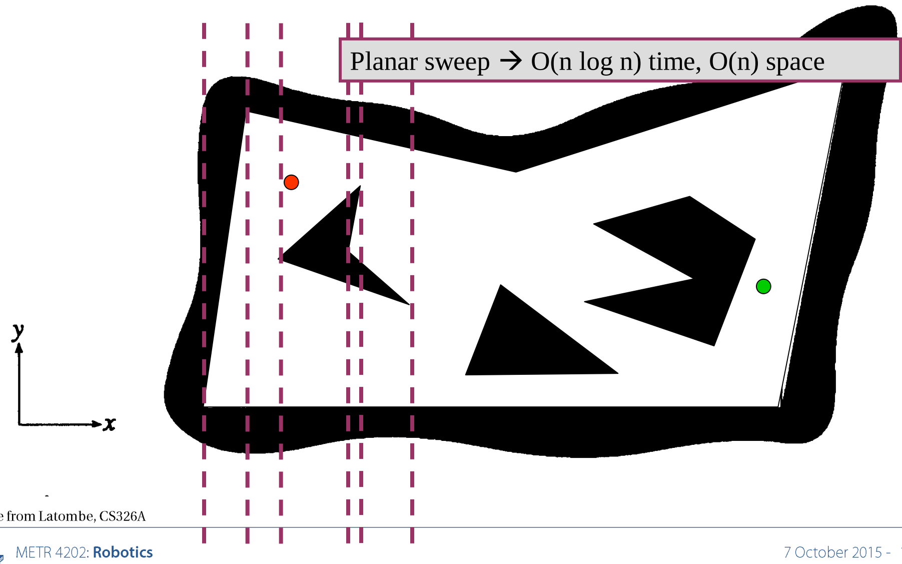
Trapezoidal decomposition



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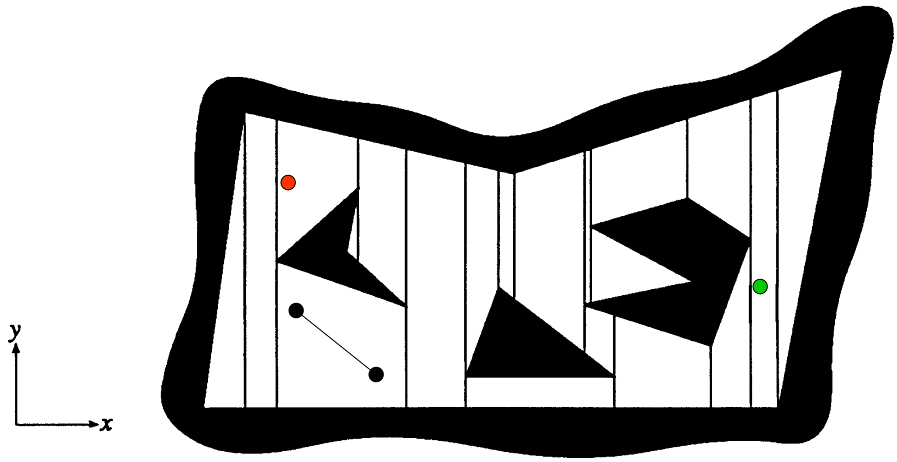
Trapezoidal decomposition



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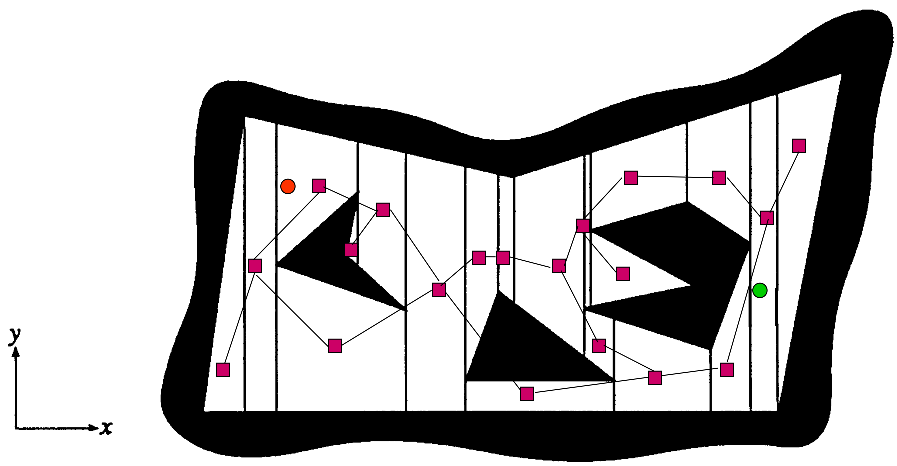
Trapezoidal decomposition



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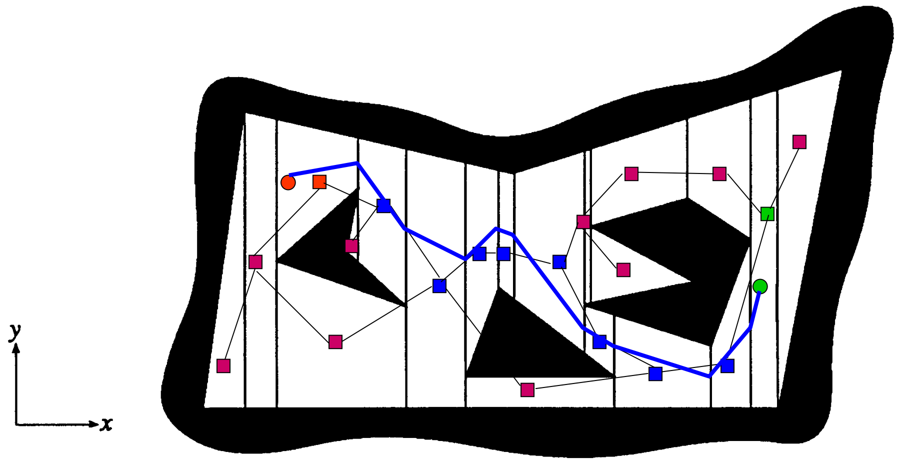
Trapezoidal decomposition



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III. Roadmap Methods

- **Visibility graph**
- **Voronoi diagram**
- Silhouette
  First complete general method that applies to spaces of any dimension and is singly exponential in # of dimensions [Canny, 87]
- **Probabilistic roadmaps (PRMS) and Rapidly-exploring Randomized Trees (RRTs)**

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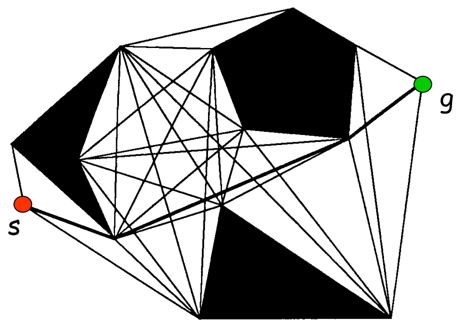
Roadmap Methods

- **Visibility graph**
  Introduced in the Shakey project at SRI in the late 60s.
  Can produce shortest paths in 2-D configuration spaces



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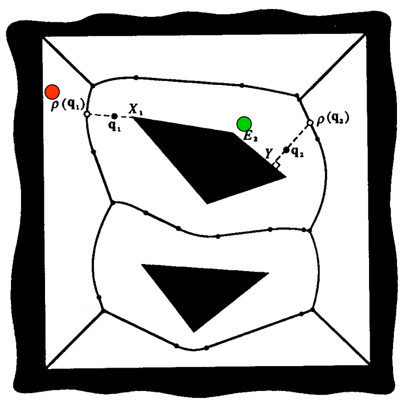
Roadmap Methods

- Voronoi diagram
  Introduced by Computational Geometry researchers.
  Generate paths that maximizes clearance.

  O(n log n) time
  O(n) space



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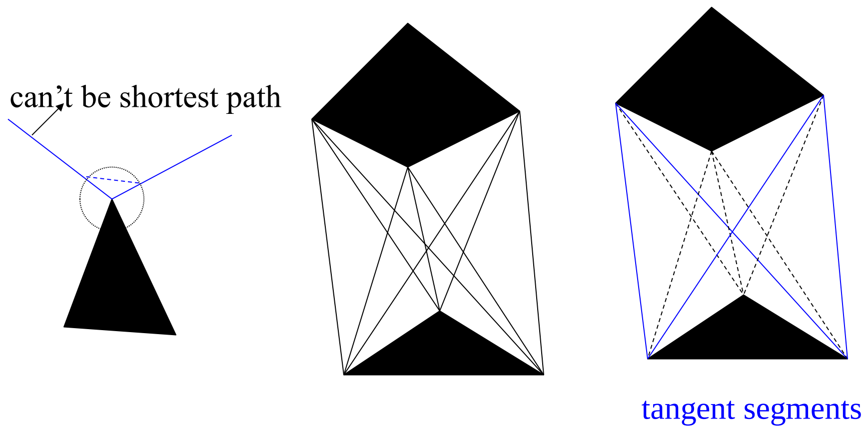
II.Visibility Graph



→ Eliminate concave obstacle vertices

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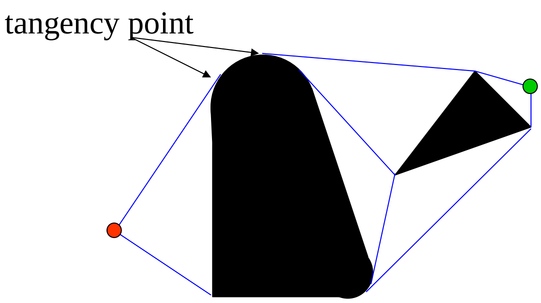
Generalized (Reduced) -- Visibility Graph



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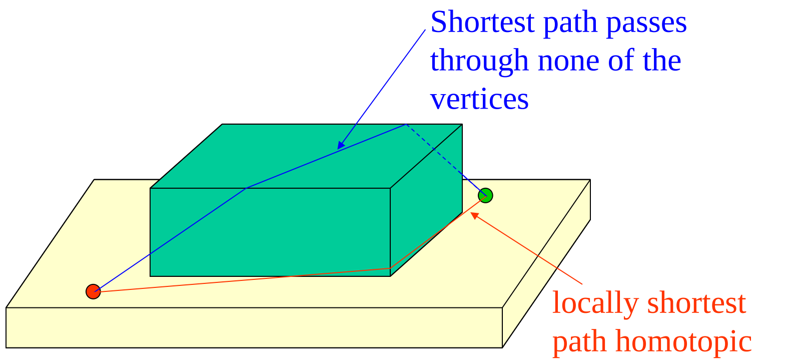
Three-Dimensional Space



Computing the shortest collision-free path in a polyhedral space is NP-hard

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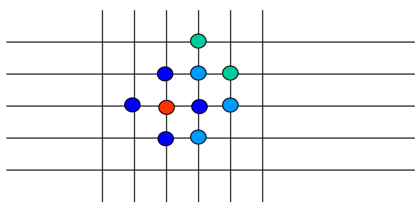
Sketch of Grid Algorithm (with best-first search)

- Place regular grid G over space
- Search G using best-first search algorithm with potential as heuristic function



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Simple Algorithm (for Visibility Graphs)

- Install all obstacles vertices in VG, plus the start and goal positions
- For every pair of nodes u, v in VG

        If segment(u,v) is an obstacle edge then
                insert (u,v) into VG
        else
        for every obstacle edge e
                if segment(u,v) intersects e
                        then go up to segment
    insert (u,v) into VG

- Search VG using A*

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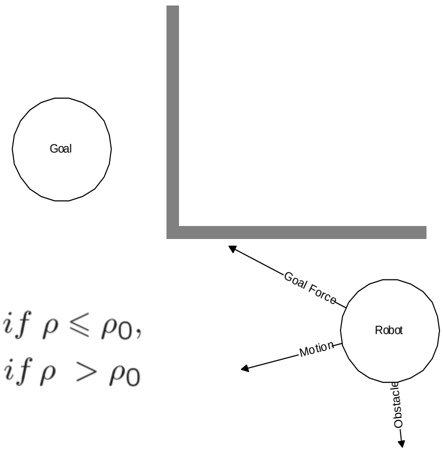
IV. Potential Field Methods

- Approach initially proposed for real-time collision avoidance [Khatib, 86]

$$F_{Goal} = -k_p(x - x_{Goal})$$

$$F_{Obstacle} = \begin{cases} \eta \left(\frac{1}{\rho} - \frac{1}{\rho_0}\right) \frac{1}{\rho^2} \frac{\partial \rho}{\partial x} & if\ \rho \leqslant \rho_0, \\ 0 & if\ \rho > \rho_0 \end{cases}$$



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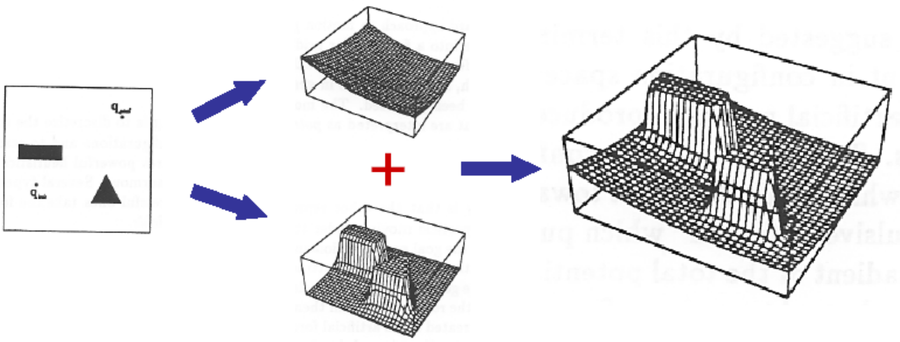
Attractive and Repulsive fields



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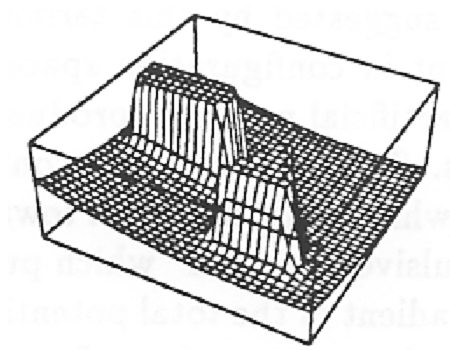
Local-Minimum Issue



- Perform best-first search (possibility of combining with approximate cell decomposition)
- Alternate descents and random walks
- Use local-minimum-free potential (navigation function)

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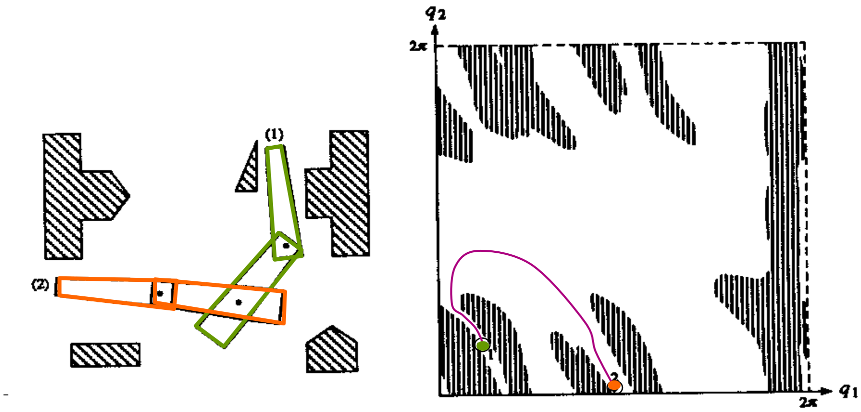
Configuration Space



- A robot configuration is a specification of the positions of all robot points relative to a fixed coordinate system
- Usually a configuration is expressed as a “vector” of position/orientation parameters

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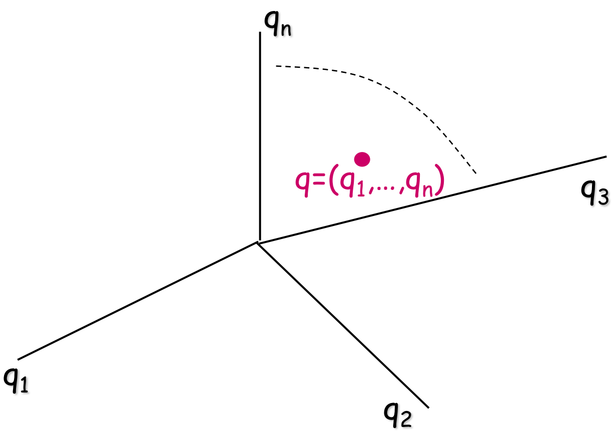
Motion Planning in C-Space



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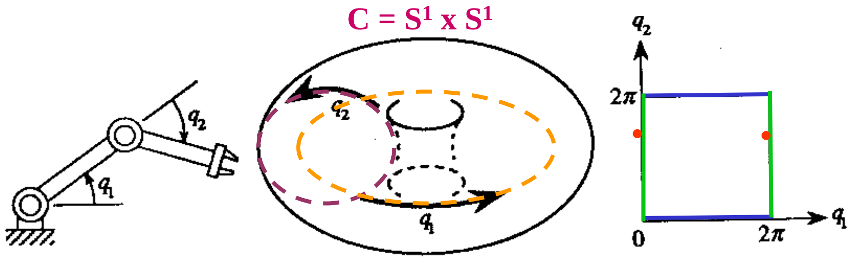
Configuration Space of a Robot

- Space of all its possible configurations
- But the topology of this space is usually not that of a Cartesian space



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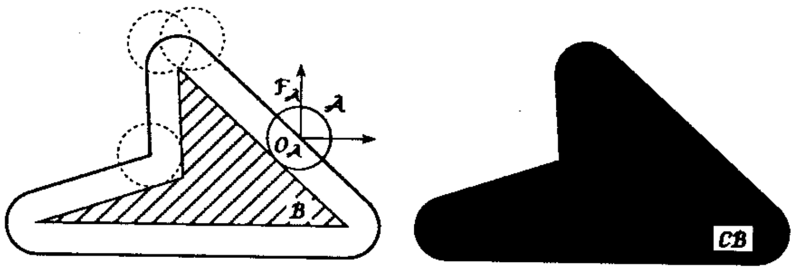
Disc Robot in 2-D Workspace



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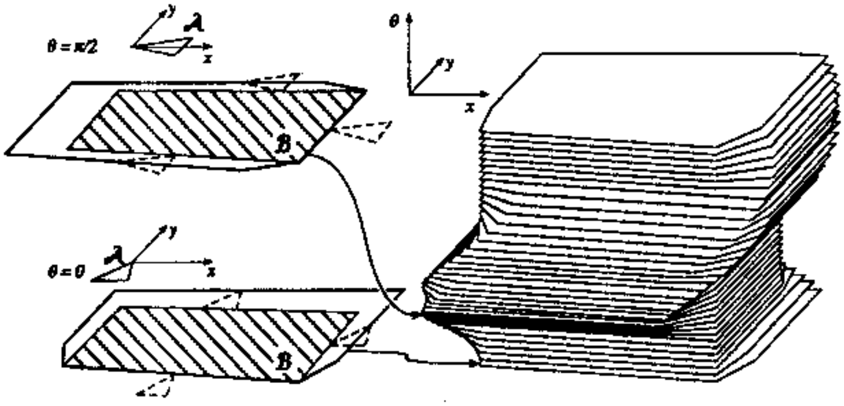
Rigid Robot Translating and Rotating in 2-D



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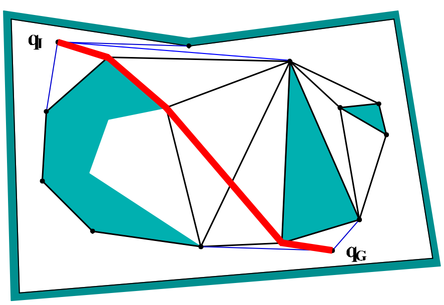
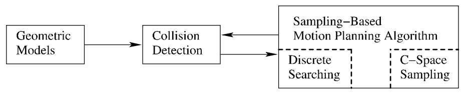
Geometric Planning Methods



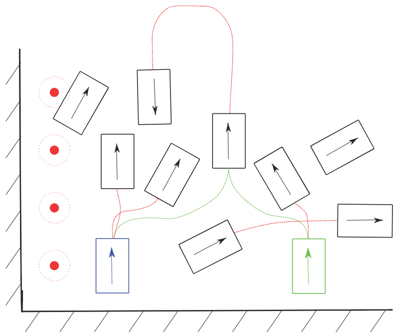
- Several Geometric Methods:
  - Vertical (Trapezoidal) Cell Decomposition
  - **Roadmap Methods**
    - **Cell (Triangular) Decomposition**
    - Visibility Graphs
    - Veroni Graphs

Artwork from LaValle, Ch. 6

Sample-Based Motion Planning



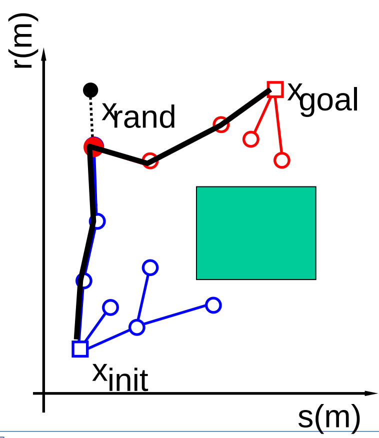
- PRMs



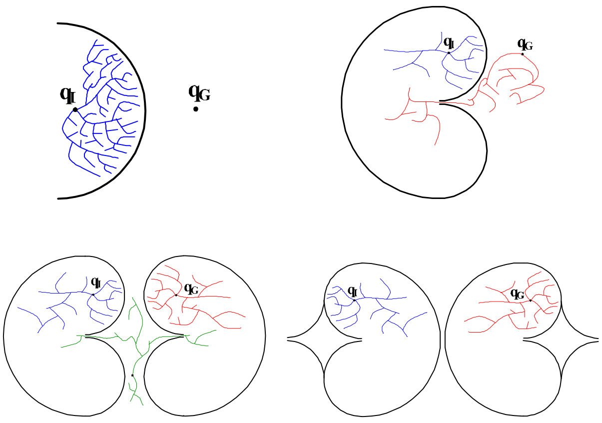
- RRTs

Artwork based on LaValle, Ch. 5

Rapidly Exploring Random Trees (RRT)

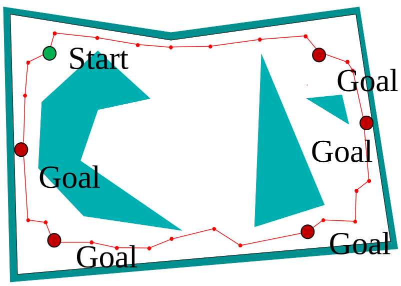


Sampling and the "Bug Trap" Problem



Artwork based on LaValle, Ch. 5

Multiple Points & Sequencing



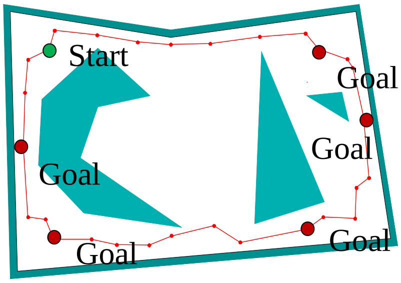
- Sequencing
  - Determining the "best" order to go in

➔ Travelling Salesman Problem

A salesman has to visit each city on a given list exactly once. In doing this, he **starts** from his home city and in the **end he has to return to his home** city. It is plausible for him to select the order in which he visits the cities so that the **total of the distances travelled** in his tour is as small as possible.

Artwork based on LaValle, Ch. 6

Travelling Salesman Problem

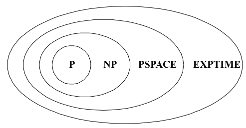


- Given a $n \times n$ distance matrix $\mathbf{C}=(c_{ij})$

- Minimize:

$$c(\pi) = \sum_{i=1}^{n} c_{i\pi(i)}$$

- Note that this problem is NP-Hard

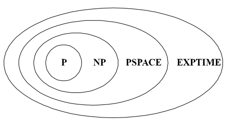


➔ BUT, Special Cases are Well-Solvable!

Artwork based on LaValle, Ch. 6

Travelling Salesman Problem [2]

- This problem is NP-Hard



➔ BUT,
Special Cases are
Well-Solvable!

For the Euclidean case
(where the points are on the 2D Euclidean plane) :

- The shortest TSP tour does not intersect itself, and thus geometry makes the problem somewhat easier.
- If all cities lie on the boundary of a convex polygon, the optimal tour is a cyclic walk along the boundary of the polygon (in clockwise or counterclockwise direction).

***The k*-line TSP**

- The a special case where the cities lie on *k* parallel (or almost parallel) lines in the Euclidean plane.
- EG: Fabrication of printed circuit boards
- Solvable in $\mathbf{O}(n^3)$ time by Dynamic Programming (Rote's algorithm)

The necklace TSP

- The special Euclidean TSP case where there exist *n* circles around the *n* cities such that every cycle intersects exactly two adjacent circles