Robot Sensing: Features & Detection (using Computer Vision)

"What's this?"

3 8 1 7 6 7 4 7 8 3 5 9 5 3 6 3 7 4 4 6 9 3 8 7 9 0 3 6 3 2 6 6 5 6 0 3 4 2 6 8 3 8 1...
7 6 7 4 7 8 3 5 9 5 3 6 3 7 4 4 6 9 3 8 7 9 0 3 6 3 2 6 6 5 6 0 3 4 2 6 8

– SIFT

METR 4202: Advanced Control & **Robotics**

Dr Surya Singh -- Lecture # 7

September 10, 2014

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Schedule

Week	Date	Lecture (W: 11:10-12:40, 24-402)
1	30-Jul	Introduction
2	6-Aug	Representing Position & Orientation & State (Frames, Transformation Matrices & Affine Transformations)
3	13-Aug	Robot Kinematics (& Ekka Day)
4	20-Aug	Robot Dynamics & Control
5	27-Aug	Robot Motion
6	3-Sep	Robot Sensing: Perception & Multiple View Geometry
7	10-Sep	**Robot Sensing: Features & Detection using Computer Vision**
8	17-Sep	Navigation & Localization (+ Prof. M. Srinivasan)
9	24-Sep	Motion Planning + Control
	1-Oct	*Study break*
10	8-Oct	State-Space Modelling
11	15-Oct	Shaping the Dynamic Response
12	22-Oct	Linear Observers & LQR
13	29-Oct	Applications in Industry & Course Review

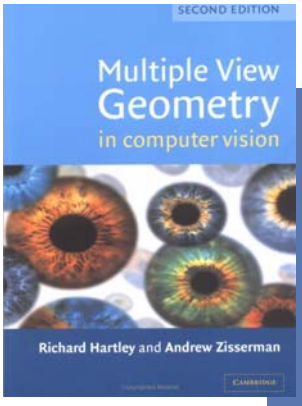
Dynamixel Kit

http://www.tribotix.com/Products/Tribotix/Kits/UQ_Kits.htm

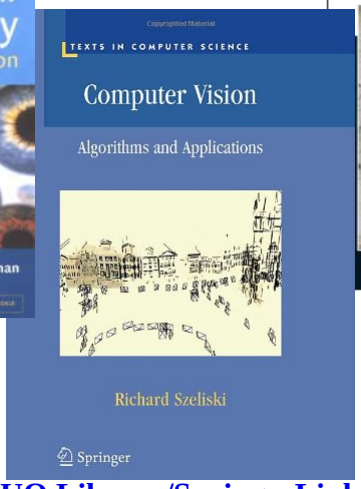
Kit includes:

Index	Part	Quantity
1	FP04-F1 Angles Hinge Bracket	2
2	FP04-F2 Stnd Hinge Bracket	4
3	FP04-F3 Bottom Bracket	5
4	FP04-F4 Large Hinge Bracket	2
5	FP04-F5 Wide Hinge Bracket	2
6	FP04-F6 Side Bracket	2
7	FP04-F7 Back Bracket	2
8	BNS-10 Bioloid Screw Set	1
9	Cable-3P Robot Cable-3P 200mm	1
10	SMPS2Dynamixel SMPS2Dynamixel	1
11	USB2Dynamixel USB2Dynamixel	1
12	AX-12A DYNAMIXEL AX-12A	3
13	DYNAMIXEL MX-12W	1

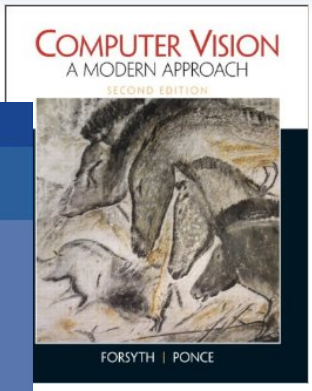
Reference Material



UQ Library
(ePDF)



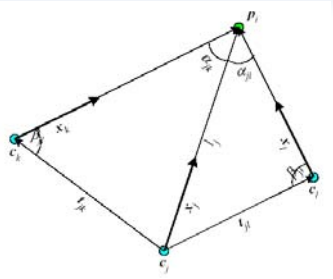
UQ Library/SpringerLink



UQ Library
(Hardcopy)

Essential matrix

- Co-planarity constraint:
  - $x' \approx R x + t$
  - $[t]\times x' \approx [t]\times R x$
  - $x' [t]\times x' \approx x' [t]\times R x$
  - $x' E x = 0$ with $E = [t]\times R$



- Solve for **E** using least squares (SVD)
- t is the least singular vector of E
- R obtained from the other two s.v.s

From Szeliski, *Computer Vision: Algorithms and Applications*

Fundamental matrix

- Camera calibrations are unknown
- $x' F x = 0$ with $F = [e]\times H = K'[t]\times R K^{-1}$
- Solve for F using least squares (SVD)
  - re-scale (xi, xi') so that $|xi| \approx 1/2$ [Hartley]
- e (epipole) is still the least singular vector of F
- H obtained from the other two s.v.s
- "plane + parallax" (projective) reconstruction
- use self-calibration to determine **K** [Pollefeys]

From Szeliski, *Computer Vision: Algorithms and Applications*

Fundamental Matrix

- The fundamental matrix is the algebraic representation of epipolar geometry.

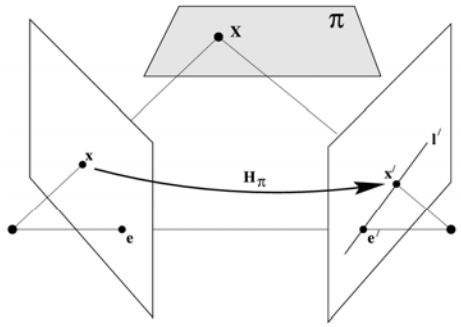


Fig. 9.5. *A point* $\mathbf{x}$ *in one image is transferred via the plane* π *to a matching point* $\mathbf{x}'$ *in the second image. The epipolar line through* $\mathbf{x}'$ *is obtained by joining* $\mathbf{x}'$ *to the epipole* $\mathbf{e}'$. *In symbols one may write* $\mathbf{x}' = \mathtt{H}_\pi\mathbf{x}$ *and* $\mathbf{l}' = [\mathbf{e}']_\times\mathbf{x}' = [\mathbf{e}']_\times\mathtt{H}_\pi\mathbf{x} = \mathtt{F}\mathbf{x}$ *where* $\mathtt{F} = [\mathbf{e}']_\times\mathtt{H}_\pi$ *is the fundamental matrix.*

Fundamental Matrix Example

- Suppose the camera matrices are those of a calibrated stereo rig with the world origin at the first camera

$$\mathtt{P} = \mathtt{K}[\mathtt{I} \mid \mathbf{0}] \qquad \mathtt{P}' = \mathtt{K}'[\mathtt{R} \mid \mathbf{t}].$$

- Then:

$$\mathtt{P}^+ = \begin{bmatrix} \mathtt{K}^{-1} \\ \mathbf{0}^\mathsf{T} \end{bmatrix} \qquad \mathbf{C} = \begin{pmatrix} \mathbf{0} \\ 1 \end{pmatrix}$$

- Epipoles are at:

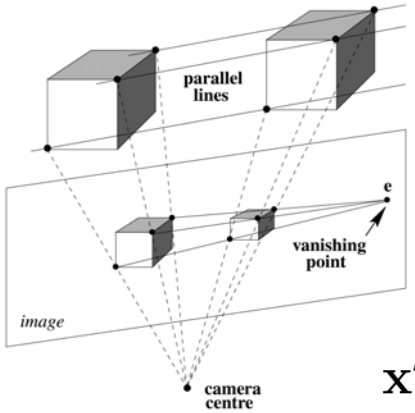
$$\therefore \qquad \mathbf{e} = \mathtt{P}\begin{pmatrix} -\mathtt{R}^\mathsf{T}\mathbf{t} \\ 1 \end{pmatrix} = \mathtt{K}\mathtt{R}^\mathsf{T}\mathbf{t} \qquad \mathbf{e}' = \mathtt{P}'\begin{pmatrix} \mathbf{0} \\ 1 \end{pmatrix} = \mathtt{K}'\mathbf{t}.$$

$$\mathtt{F} = [\mathbf{e}']_\times\mathtt{K}'\mathtt{R}\mathtt{K}^{-1} = \mathtt{K}'^{-\mathsf{T}}[\mathbf{t}]_\times\mathtt{R}\mathtt{K}^{-1} = \mathtt{K}'^{-\mathsf{T}}\mathtt{R}[\mathtt{R}^\mathsf{T}\mathbf{t}]_\times\mathtt{K}^{-1} = \mathtt{K}'^{-\mathsf{T}}\mathtt{R}\mathtt{K}^\mathsf{T}[\mathbf{e}]_\times$$

Summary of fundamental matrix properties

- $\mathtt{F}$ is a rank 2 homogeneous matrix with 7 degrees of freedom.
- **Point correspondence**: If $\mathbf{x}$ and $\mathbf{x}'$ are corresponding image points, then $\mathbf{x}'^{\mathsf{T}}\mathtt{F}\mathbf{x} = 0$.
- **Epipolar lines**:
  - $\mathbf{l}' = \mathtt{F}\mathbf{x}$ is the epipolar line corresponding to $\mathbf{x}$.
  - $\mathbf{l} = \mathtt{F}^{\mathsf{T}}\mathbf{x}'$ is the epipolar line corresponding to $\mathbf{x}'$.
- **Epipoles**:
  - $\mathtt{F}\mathbf{e} = \mathbf{0}$.
  - $\mathtt{F}^{\mathsf{T}}\mathbf{e}' = \mathbf{0}$.
- **Computation from camera matrices** $\mathtt{P}, \mathtt{P}'$:
  - General cameras, $\mathtt{F} = [\mathbf{e}']_\times \mathtt{P}'\mathtt{P}^+$, where $\mathtt{P}^+$ is the pseudo-inverse of $\mathtt{P}$, and $\mathbf{e}' = \mathtt{P}'\mathbf{C}$, with $\mathtt{P}\mathbf{C} = \mathbf{0}$.
  - Canonical cameras, $\mathtt{P} = [\mathtt{I} \mid \mathbf{0}]$, $\mathtt{P}' = [\mathtt{M} \mid \mathbf{m}]$, $\mathtt{F} = [\mathbf{e}']_\times \mathtt{M} = \mathtt{M}^{-\mathsf{T}}[\mathbf{e}]_\times$, where $\mathbf{e}' = \mathbf{m}$ and $\mathbf{e} = \mathtt{M}^{-1}\mathbf{m}$.
  - Cameras not at infinity $\mathtt{P} = \mathtt{K}[\mathtt{I} \mid \mathbf{0}]$, $\mathtt{P}' = \mathtt{K}'[\mathtt{R} \mid \mathbf{t}]$, $\mathtt{F} = \mathtt{K}'^{-\mathsf{T}}[\mathbf{t}]_\times \mathtt{R}\mathtt{K}^{-1} = [\mathtt{K}'\mathbf{t}]_\times \mathtt{K}'\mathtt{R}\mathtt{K}^{-1} = \mathtt{K}'^{-\mathsf{T}}\mathtt{R}\mathtt{K}^{\mathsf{T}}[\mathtt{K}\mathtt{R}^{\mathsf{T}}\mathbf{t}]_\times$.

Fundamental Matrix & Motion

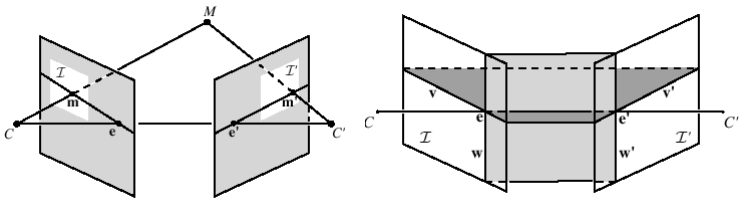


$$\mathbf{x}'^{\mathsf{T}}\mathtt{F}\mathbf{x} = 0.$$

- Under a pure translational camera motion, 3D points appear to slide along parallel rails. The images of these parallel lines intersect in a vanishing point corresponding to the translation direction. The epipole **e** is the vanishing point.

Rectification

- Project each image onto same plane, which is parallel to the epipole
- Resample lines (and shear/stretch) to place lines in correspondence, and minimize distortion

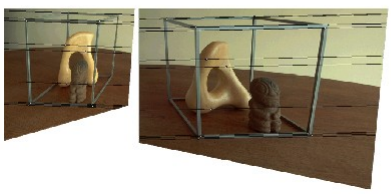


- [Zhang and Loop, MSR-TR-99-21]

Slide from Szeliski, *Computer Vision: Algorithms and Applications*

Rectification

(a) Original image pair overlayed with several epipolar lines.

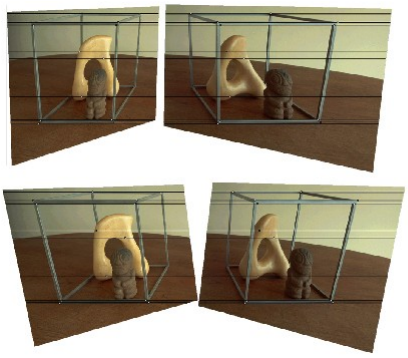


(b) Image pair transformed by the specialized projective mapping $\mathbf{H}_p$ and $\mathbf{H}'_p$. Note that the epipolar lines are now parallel to each other in each image.

BAD!

Slide from Szeliski, *Computer Vision: Algorithms and Applications*

Rectification



(c) Image pair transformed by the similarity $\mathbf{H}_r$ and $\mathbf{H}'_r$. Note that the image pair is now rectified (the epipolar lines are horizontally aligned).

(d) Final image rectification after shearing transform $\mathbf{H}_a$ and $\mathbf{H}'_a$. Note that the image pair remains rectified, but the horizontal distortion is reduced.

GOOD!

Slide from Szeliski, *Computer Vision: Algorithms and Applications*

Finding correspondences

- Apply feature matching criterion (e.g., correlation or Lucas-Kanade) at all pixels simultaneously
- Search only over epipolar lines (many fewer candidate positions)

Slide from Szeliski, *Computer Vision: Algorithms and Applications*

Image registration (revisited)

- How do we determine correspondences?
  - block matching or SSD (sum squared differences)

d is the disparity (horizontal motion)

$$E(x, y; d) = \sum_{(x',y') \in N(x,y)} [I_L(x'+d, y') - I_R(x', y')]^2$$

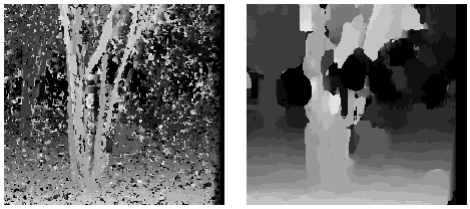
- How big should the neighborhood be?

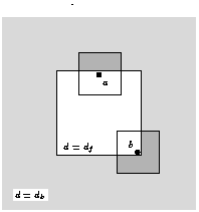
Slide from Szeliski, *Computer Vision: Algorithms and Applications*

Neighborhood size

- Smaller neighborhood: more details
- Larger neighborhood: fewer isolated mistakes

W



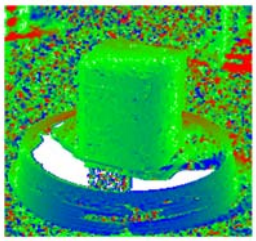


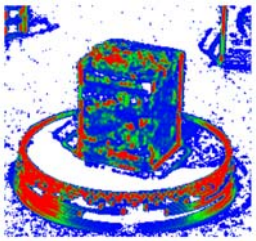
Slide from Szeliski, *Computer Vision: Algorithms and Applications*

Stereo: certainty modeling

- Compute certainty map from correlations


input


depth map


certainty map

How to get Matching Points? **Features**

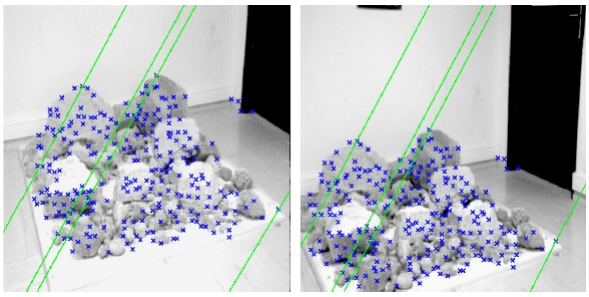
- ~~Colour~~
- Corners
- Edges
- Lines
- Statistics on Edges: SIFT, SURF, ORB…

In OpenCV: The following detector types are supported:

- "FAST" – FastFeatureDetector
- "STAR" – StarFeatureDetector
- "SIFT" – SIFT (nonfree module)
- "SURF" – SURF (nonfree module)
- "ORB" – ORB
- "BRISK" – BRISK
- "MSER" – MSER
- "GFTT" – GoodFeaturesToTrackDetector
- "HARRIS" – GoodFeaturesToTrackDetector with Harris detector enabled
- "Dense" – DenseFeatureDetector
- "SimpleBlob" – SimpleBlobDetector

Feature-based stereo

- Match “corner” (interest) points



- Interpolate complete solution

Slide from Szeliski, *Computer Vision: Algorithms and Applications*

Features -- Colour Features

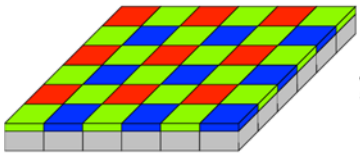
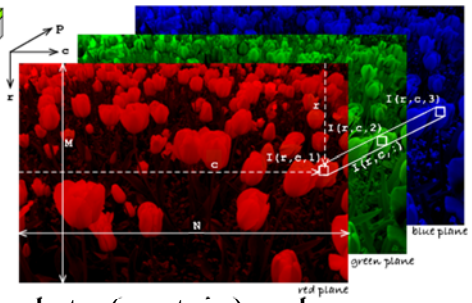


Fig: Ch. 10, *Robotics Vision and Control*

Bayer Patterns



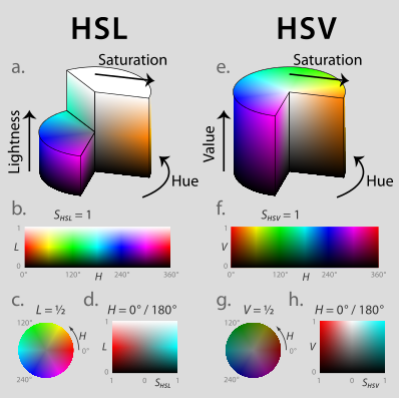
- RGB is **NOT** an absolute (metric) colour space

Also!

- **RGB** (display or additive colour) does not map to **CYMK** (printing or subtractive colour) without calibration
- **Y-Cr-Cb** or **HSV** does not solve this either

Colour Spaces

- HSV



Source: Wikipedia – HSV and YCrCb

- YCrCb

➔Gamma Corrected Luma (Y) + Chrominance

➔BW → Colour TVs : Just add the Chrominance

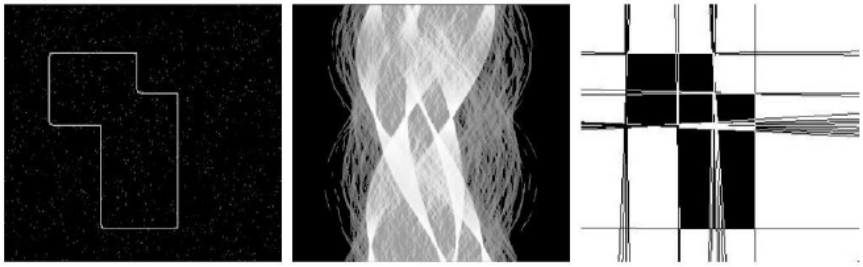
➔γ Correction: CRTs γ=2.2-2.5

$$\begin{aligned} Y' &= 16 + (65.481 \cdot R' + 128.553 \cdot G' + 24.966 \cdot B') \\ C_B &= 128 + (-37.797 \cdot R' - 74.203 \cdot G' + 112.0 \cdot B') \\ C_R &= 128 + (112.0 \cdot R' - 93.786 \cdot G' - 18.214 \cdot B') \end{aligned}$$

How to get the Features? Still MANY Ways

- Canny edge detector:

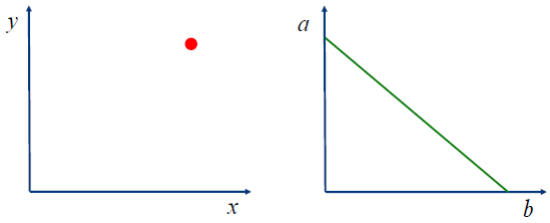
Hough Transform



- Uses a voting mechanism
- Can be used for other lines and shapes (not just straight lines)

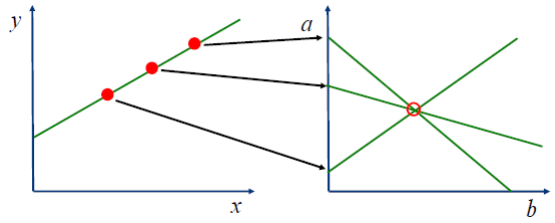
Hough Transform: Voting Space

$$y = ax + b \qquad a = -\frac{1}{x}b + \frac{y}{x}$$



- Count the number of lines that can go through a point and move it from the "*x*-*y*" plane to the "a-b" plane
- There is only a one-"infinite" number (a line!) of solutions (not a two-"infinite" set – a plane)

Hough Transform: Voting Space



- In practice, the polar form is often used

$$a = x \cos a + y \sin b$$

- This avoids problems with lines that are nearly vertical

Hough Transform: Algorithm

1. Quantize the parameter space appropriately.

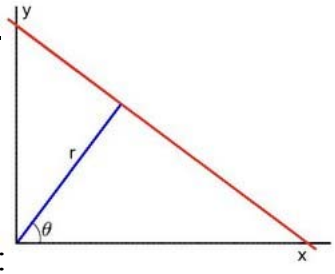
2. Assume that each cell in the parameter space is an accumulator. Initialize all cells to zero.

3. For each point (x,y) in the (visual & range) image space, increment by 1 each of the accumulators that satisfy the equation.

4. Maxima in the accumulator array correspond to the parameters of model instances.

Line Detection – Hough Lines [1]

- A line in an image can be expressed as two variables:
  - **Cartesian coordinate system:** m,b
  - **Polar coordinate system:** r, θ
    ➔ **<u>avoids problems with vert. lines</u>**



y=**m**x+**b** ➔

$$y = \left(-\frac{\cos\theta}{\sin\theta}\right)x + \left(\frac{r}{\sin\theta}\right)$$

- For each point (x_1, y_1) we can write:

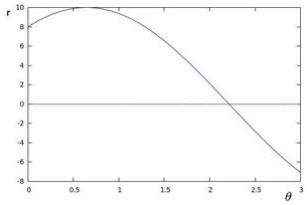
$$r = x_1\cos\theta + y_1\sin\theta$$

- Each pair (r,θ) represents a line that passes through (x_1, y_1)

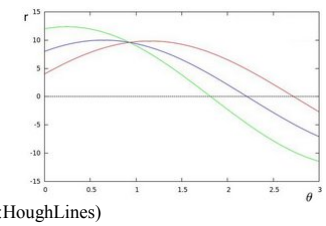
See also OpenCV documentation (cv::HoughLines)

Line Detection – Hough Lines [2]

- Thus a given point gives a sinusoid



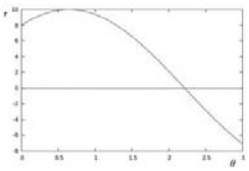
- Repeating for all points on the image

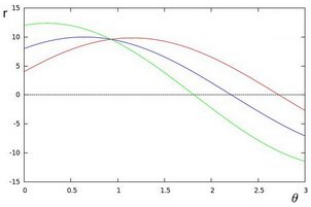


See also OpenCV documentation (cv::HoughLines)

Line Detection – Hough Lines [3]

- Thus a given point gives a sinusoid





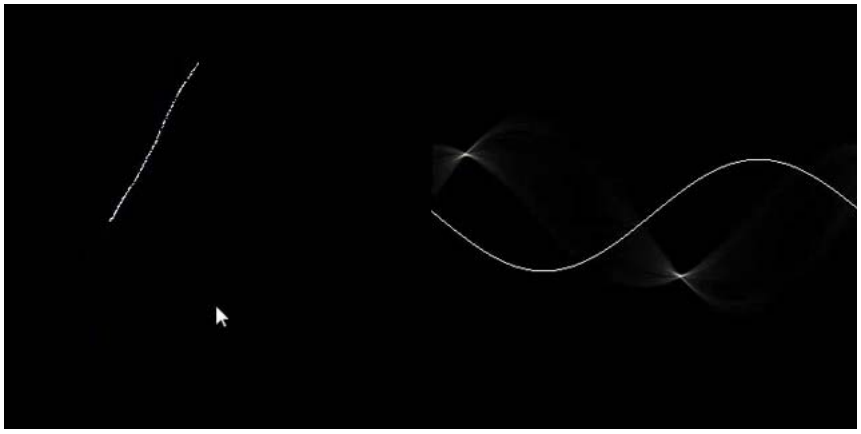
- Repeating for all points on the image

- NOTE that an intersection of sinusoids represents (**a point**) represents **a line** in which pixel points lay.

➔ Thus, a line can be *detected* by finding the number of Intersections between curves

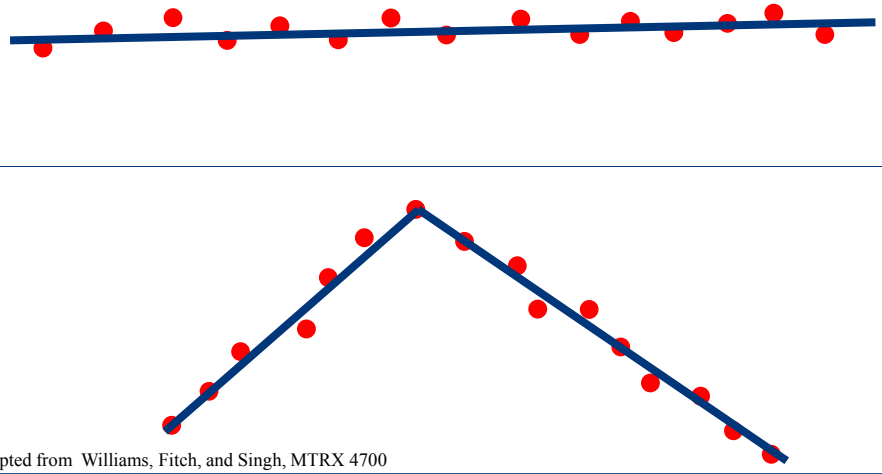
See also OpenCV documentation (cv::HoughLines)

"Cool Robotics Share" -- Hough Transform



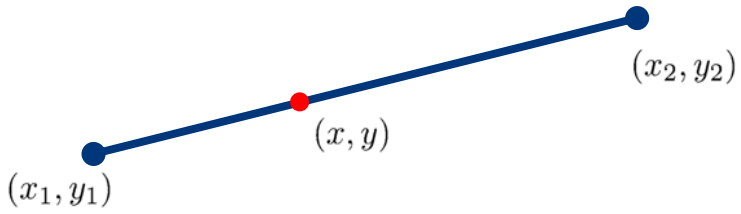
- http://www.activovision.com/octavi/doku.php?id=hough_transform

Line Extraction and Segmentation



Adopted from Williams, Fitch, and Singh, MTRX 4700

Line Formula



$$\frac{y - y_1}{x - x_1} = \frac{y_2 - y_1}{x_2 - x_1}$$

$$\mathbf{y} = m\mathbf{x} + b$$

Adopted from Williams, Fitch, and Singh, MTRX 4700

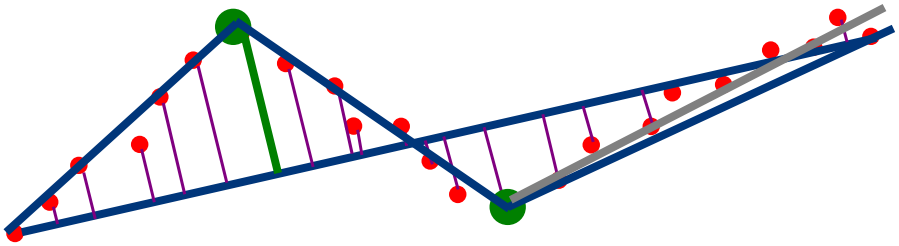
Line Estimation

Least squares minimization of the line:

- Line Equation: $\mathbf{y} - m\mathbf{x} - b = 0$
- Error in Fit: $\sum_i (y_i - mx_i - b)^2$
- Solution: $\begin{pmatrix} \bar{x}\bar{y} \\ \bar{y} \end{pmatrix} = \begin{pmatrix} \overline{x^2} & \bar{x} \\ \bar{x} & 1 \end{pmatrix} \begin{pmatrix} m \\ b \end{pmatrix}$

Adopted from Williams, Fitch, and Singh, MTRX 4700

Line Splitting / Segmentation



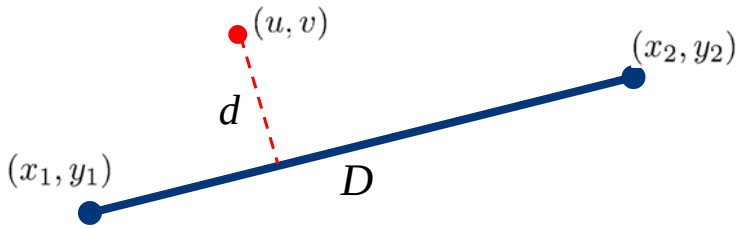
- What about corners?

➔Split into multiple lines (via expectation maximization)

1. Expect (assume) a number of lines N (say 3)
2. Find "breakpoints" by finding nearest neighbours upto a threshold or simply at random (RANSAC)
3. How to know N? (Also RANSAC)

Adopted from Williams, Fitch, and Singh, MTRX 4700

⊥ of a Point from a Line Segment



$$r = u(y_1 - y_2) + v(x_2 - x_1) + y_2x_1 - y_1x_2$$

$$d = \frac{r}{D}$$

Adopted from Williams, Fitch, and Singh, MTRX 4700

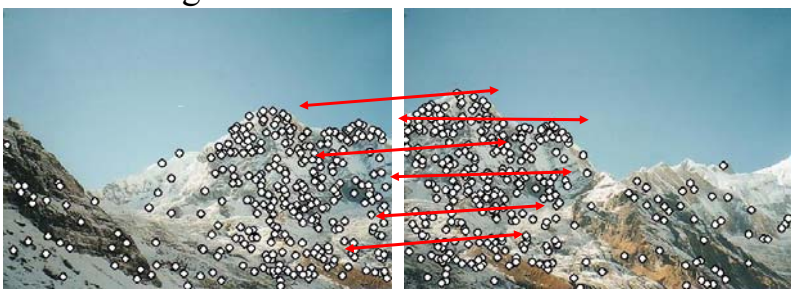
Edge Detection

- Canny edge detector:
  - Pepsi Sequence:

Image Data: http://www.cs.brown.edu/~black/mixtureOF.html and Szeliski, CS223B-L9
See also: Use of Temporal information to aid segmentation:
http://www.cs.toronto.edu/~babalex/SpatiotemporalClosure/supplementary_material.html

Why extract features?

- **Object detection**
- Robot Navigation
- Scene Recognition



- Steps:
  - Extract Features
  - Match Features

Adopted drom S. Lazebnik, Gang Hua (CS 558)

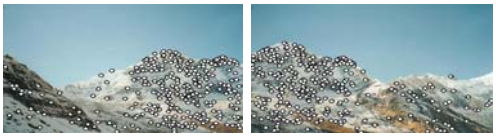
Why extract features? [2]

- Panorama stitching…
  → Step 3: Align images

Adopted from S. Lazebnik, Gang Hua (CS 558)

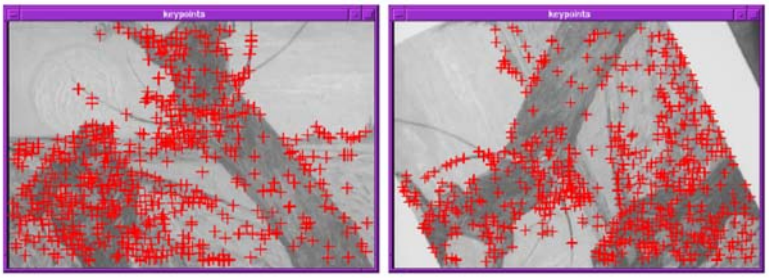
Characteristics of good features

- Repeatability
  - The same feature can be found in several images despite geometric and photometric transformations
- Saliency
  - Each feature is distinctive
- Compactness and efficiency
  - Many fewer features than image pixels
- Locality
  - A feature occupies a relatively small area of the image; robust to clutter and occlusion



Adopted from S. Lazebnik, Gang Hua (CS 558)

Finding Corners



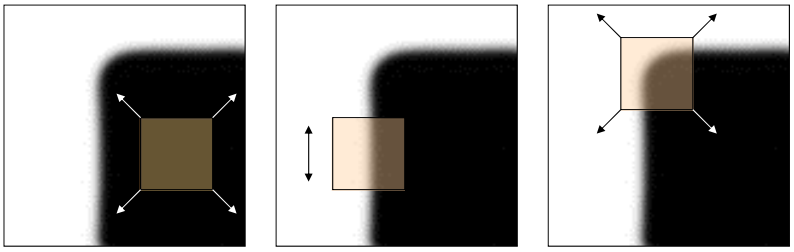
- Key property: in the region around a corner, image gradient has two or more dominant directions
- Corners are repeatable and distinctive

C.Harris and M.Stephens. "A Combined Corner and Edge Detector." *Proceedings of the 4th Alvey Vision Conference*: pages 147—151, 1988.

Adopted from S. Lazebnik, Gang Hua (CS 558)

Corner Detection: Basic Idea

- Look through a window
- Shifting a window in any direction should give a large change in intensity



"flat" region: no change in all directions

"edge": no change along the edge direction

"corner": significant change in all directions

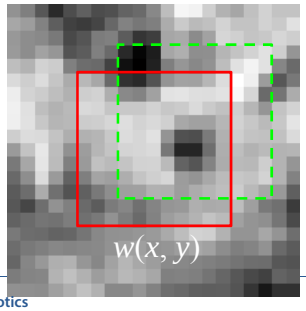
Source: A. Efros

Corner Detection: Mathematics

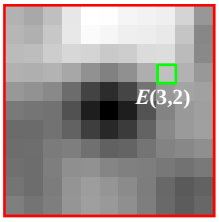
Change in appearance of window $w(x,y)$ for the shift $[u,v]$:

$$E(u,v) = \sum_{x,y} w(x,y)\left[I(x+u,y+v) - I(x,y)\right]^2$$

$I(x, y)$



$E(u, v)$



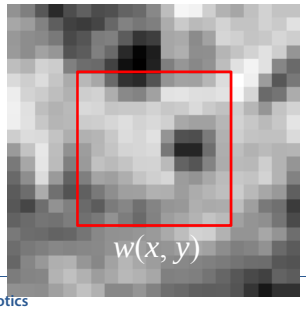
Adopted from S. Lazebnik, Gang Hua (CS 558)

Corner Detection: Mathematics

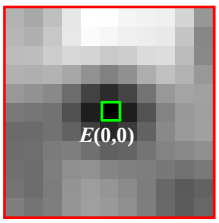
Change in appearance of window $w(x,y)$
for the shift $[u,v]$:

$$E(u,v) = \sum_{x,y} w(x,y)\left[I(x+u,y+v) - I(x,y)\right]^2$$

$I(x, y)$



$E(u, v)$



Adopted from
S. Lazebnik,
Gang Hua (CS 558)

Corner Detection: Mathematics

Change in appearance of window $w(x,y)$
for the shift $[u,v]$:

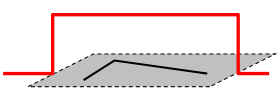
$$E(u,v) = \sum_{x,y} w(x,y)\left[I(x+u,y+v) - I(x,y)\right]^2$$

Window function

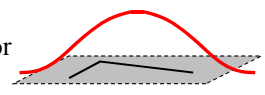
Shifted intensity

Intensity

Window function $w(x,y)$ =



or



1 in window, 0 outside

Gaussian

Adopted from
S. Lazebnik,
Gang Hua (CS 558)

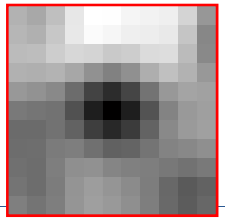
Corner Detection: Mathematics

Change in appearance of window $w(x,y)$
for the shift $[u,v]$:

$$E(u,v) = \sum_{x,y} w(x,y)\left[I(x+u,y+v) - I(x,y)\right]^2$$

We want to find out how this function behaves for small shifts

$E(u, v)$



Adopted from
S. Lazebnik,
Gang Hua (CS 558)

Corner Detection: Mathematics

Change in appearance of window $w(x,y)$
for the shift $[u,v]$:

$$E(u,v) = \sum_{x,y} w(x,y)\left[I(x+u,y+v) - I(x,y)\right]^2$$

We want to find out how this function behaves for small shifts

$$E(u,v) \approx E(0,0) + [u \;\; v]\begin{bmatrix} E_u(0,0) \\ E_v(0,0) \end{bmatrix} + \frac{1}{2}[u \;\; v]\begin{bmatrix} E_{uu}(0,0) & E_{uv}(0,0) \\ E_{uv}(0,0) & E_{vv}(0,0) \end{bmatrix}\begin{bmatrix} u \\ v \end{bmatrix}$$

Local quadratic approximation of $E(u,v)$ in the neighborhood of (0,0) is given by the *second-order Taylor expansion*:

Adopted from
S. Lazebnik,
Gang Hua (CS 558)

Corner Detection: Mathematics

$$E(u,v) = \sum_{x,y} w(x,y)\left[I(x+u,y+v) - I(x,y)\right]^2$$

Second-order Taylor expansion of $E(u,v)$ about (0,0):

$$E(u,v) \approx E(0,0) + \begin{bmatrix} u & v \end{bmatrix} \begin{bmatrix} E_u(0,0) \\ E_v(0,0) \end{bmatrix} + \frac{1}{2} \begin{bmatrix} u & v \end{bmatrix} \begin{bmatrix} E_{uu}(0,0) & E_{uv}(0,0) \\ E_{uv}(0,0) & E_{vv}(0,0) \end{bmatrix} \begin{bmatrix} u \\ v \end{bmatrix}$$

$$E_u(u,v) = \sum_{x,y} 2w(x,y)\left[I(x+u,y+v) - I(x,y)\right] I_x(x+u,y+v)$$

$$E_{uu}(u,v) = \sum_{x,y} 2w(x,y) I_x(x+u,y+v) I_x(x+u,y+v) + \sum_{x,y} 2w(x,y)\left[I(x+u,y+v) - I(x,y)\right] I_{xx}(x+u,y+v)$$

$$E_{uv}(u,v) = \sum_{x,y} 2w(x,y) I_y(x+u,y+v) I_x(x+u,y+v) + \sum_{x,y} 2w(x,y)\left[I(x+u,y+v) - I(x,y)\right] I_{xy}(x+u,y+v)$$

Adopted from S. Lazebnik, Gang Hua (CS 558)

Corner Detection: Mathematics

$$E(u,v) = \sum_{x,y} w(x,y)\left[I(x+u,y+v) - I(x,y)\right]^2$$

Second-order Taylor expansion of $E(u,v)$ about (0,0):

$$E(u,v) \approx \begin{bmatrix} u & v \end{bmatrix} \begin{bmatrix} \sum_{x,y} w(x,y) I_x^2(x,y) & \sum_{x,y} w(x,y) I_x(x,y) I_y(x,y) \\ \sum_{x,y} w(x,y) I_x(x,y) I_y(x,y) & \sum_{x,y} w(x,y) I_y^2(x,y) \end{bmatrix} \begin{bmatrix} u \\ v \end{bmatrix}$$

$$E(0,0) = 0$$

$$E_u(0,0) = 0$$

$$E_v(0,0) = 0$$

$$E_{uu}(0,0) = \sum_{x,y} 2w(x,y) I_x(x,y) I_x(x,y)$$

$$E_{vv}(0,0) = \sum_{x,y} 2w(x,y) I_y(x,y) I_y(x,y)$$

$$E_{uv}(0,0) = \sum_{x,y} 2w(x,y) I_x(x,y) I_y(x,y)$$

Adopted from S. Lazebnik, Gang Hua (CS 558)

Harris detector: Steps

- Compute Gaussian derivatives at each pixel
- Compute second moment matrix M in a Gaussian window around each pixel
- Compute corner response function R
- Threshold R
- Find local maxima of response function (nonmaximum suppression)

C.Harris and M.Stephens. "A Combined Corner and Edge Detector."
Proceedings of the 4th Alvey Vision Conference: pages 147—151, 1988.

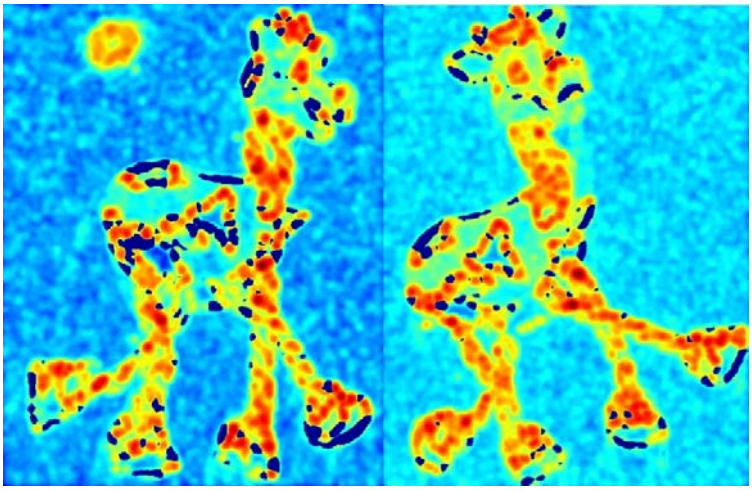
Adopted from
S. Lazebnik,
Gang Hua (CS 558)

Harris Detector: Steps

Adopted from S. Lazebnik, Gang Hua (CS 558)

Harris Detector: Steps

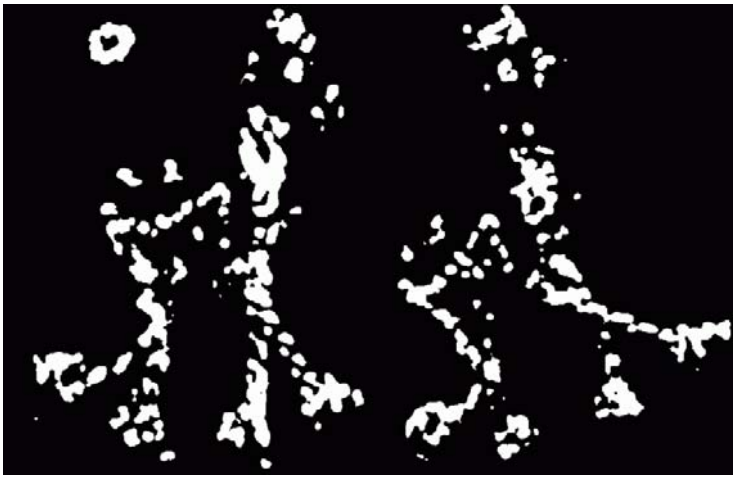
Compute corner response R



Adopted from S. Lazebnik, Gang Hua (CS 558)

Harris Detector: Steps

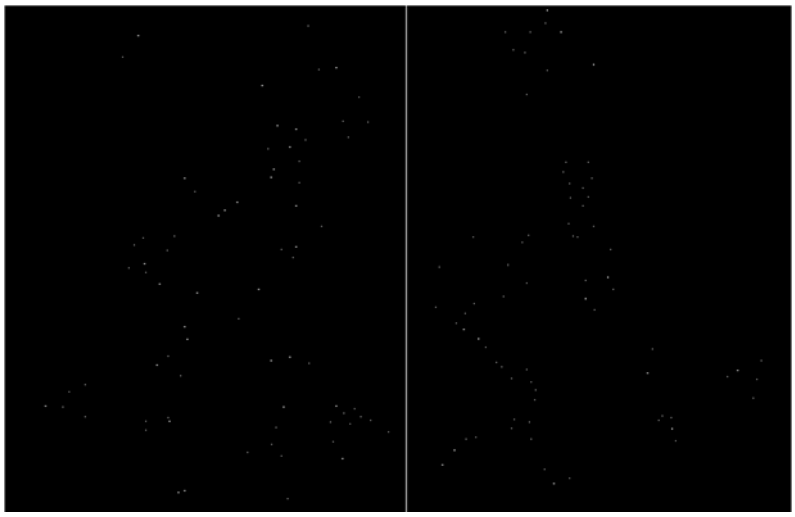
Find points with large corner response: R>threshold



Adopted from S. Lazebnik, Gang Hua (CS 558)

Harris Detector: Steps

Take only the points of local maxima of R



Adopted from S. Lazebnik, Gang Hua (CS 558)

Harris Detector: Steps

Adopted from S. Lazebnik, Gang Hua (CS 558)

Invariance and covariance

- We want corner locations to be invariant to photometric transformations and covariant to geometric transformations
  - Invariance: image is transformed and corner locations do not change
  - Covariance: if we have two transformed versions of the same image, features should be detected in corresponding locations

Adopted from S. Lazebnik, Gang Hua (CS 558)

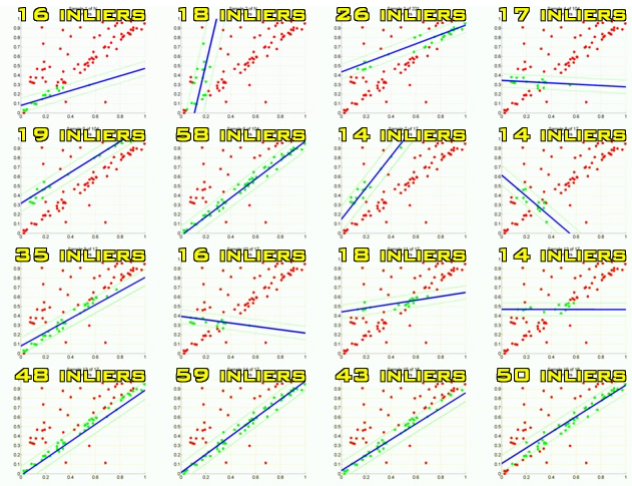
RANdom SAmple Consensus

1. Repeatedly select a small (minimal) subset of correspondences
2. Estimate a solution (in this case a the line)
3. Count the number of "inliers", $|e|<\Theta$ (for LMS, estimate med($|e|$)
4. Pick the *best* subset of inliers
5. Find a complete least-squares solution

- Related to least median squares
- See also: MAPSAC (Maximum *A Posteriori* SAmple Consensus)

From Szeliski, *Computer Vision: Algorithms and Applications*

Cool Robotics Share Time!

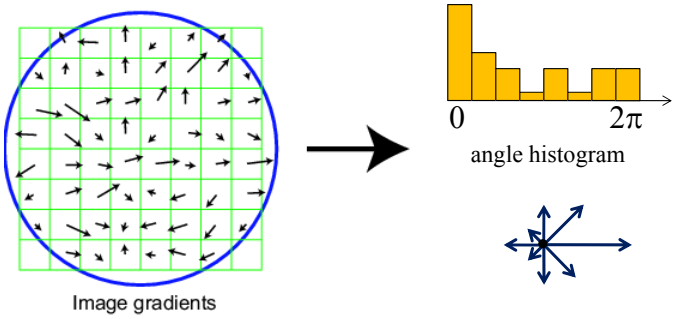


D. Wedge, *The RANSAC Song*

Scale Invariant Feature Transform

Basic idea:

- Take 16x16 square window around detected feature
- Compute edge orientation (angle of the gradient - 90°) for each pixel
- Throw out weak edges (threshold gradient magnitude)
- Create histogram of surviving edge orientations

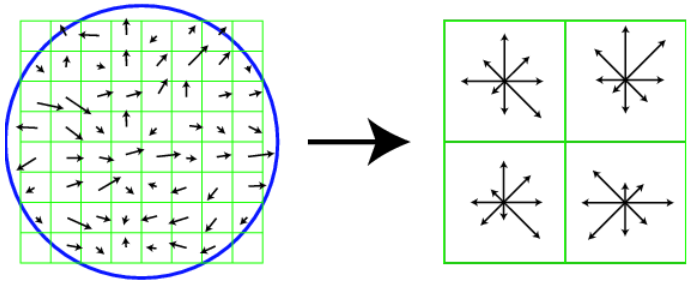


Adapted from slide by David Lowe

SIFT descriptor

Full version

- Divide the 16x16 window into a 4x4 grid of cells (2x2 case shown below)
- Compute an orientation histogram for each cell
- 16 cells * 8 orientations = 128 dimensional descriptor



Adapted from slide by David Lowe

Properties of SIFT

- Extraordinarily robust matching technique
  - Can handle changes in viewpoint
    - Up to about 60 degree out of plane rotation
  - Can handle significant changes in illumination
    - Sometimes even day vs. night (below)
  - Fast and efficient—can run in real time
  - Lots of code available
    - http://people.csail.mit.edu/albert/ladypack/wiki/index.php/Known_implementations_of_SIFT

From David Lowe and Szeliski, *Computer Vision: Algorithms and Applications*

Feature matching

- Given a feature in I_1, how to find the best match in I_2?
  1. Define distance function that compares two descriptors
  2. Test all the features in I_2, find the one with min distance

From Szeliski, *Computer Vision: Algorithms and Applications*

Feature distance

- How to define the difference between two features f_1, f_2?
  - Simple approach is SSD(f_1, f_2)
    - sum of square differences between entries of the two descriptors
    - can give good scores to very ambiguous (bad) matches

I_1 I_2

From Szeliski, *Computer Vision: Algorithms and Applications*

Feature distance

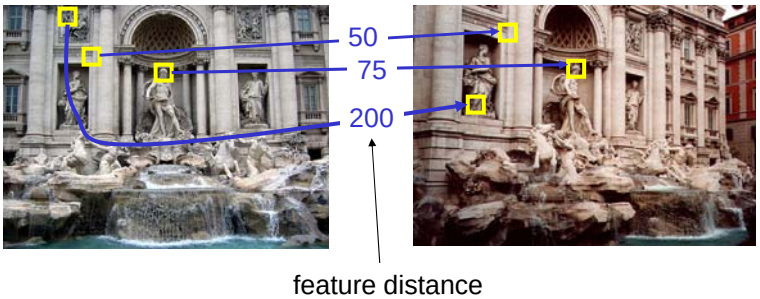
- How to define the difference between two features f_1, f_2?
  - Better approach: ratio distance = SSD(f_1, f_2) / SSD(f_1, f_2')
    - f_2 is best SSD match to f_1 in I_2
    - f_2' is 2nd best SSD match to f_1 in I_2
    - gives small values for ambiguous matches

I_1 I_2

From Szeliski, *Computer Vision: Algorithms and Applications*

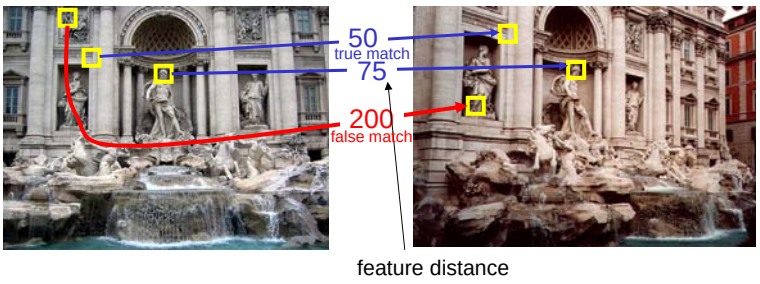
Evaluating the results

- How can we measure the performance of a feature matcher?



From Szeliski, *Computer Vision: Algorithms and Applications*

True/false positives



- The distance threshold affects performance
  - True positives = # of detected matches that are correct
    - Suppose we want to maximize these—how to choose threshold?
  - False positives = # of detected matches that are incorrect
    - Suppose we want to minimize these—how to choose threshold?

From Szeliski, *Computer Vision: Algorithms and Applications*

Levenberg-Marquardt

- Iterative non-linear least squares [Press'92]
  - Linearize measurement equations

$$\hat{u}_i = f(\mathbf{m}, \mathbf{x}_i) + \frac{\partial f}{\partial \mathbf{m}} \Delta \mathbf{m}$$
$$\hat{v}_i = g(\mathbf{m}, \mathbf{x}_i) + \frac{\partial g}{\partial \mathbf{m}} \Delta \mathbf{m}$$

  - Substitute into log-likelihood equation: quadratic cost function in Dm

$$\sum_i \sigma_i^{-2} (\hat{u}_i - u_i + \frac{\partial f}{\partial \mathbf{m}} \Delta \mathbf{m})^2 + \cdots$$

From Szeliski, *Computer Vision: Algorithms and Applications*

Levenberg-Marquardt

- What if it doesn't converge?
  - Multiply diagonal by (1 + l), increase l until it does
  - Halve the step size Dm (my favorite)
  - Use line search
  - Other ideas?
- Uncertainty analysis: covariance S = A-1
- Is maximum likelihood the best idea?
- How to start in vicinity of global minimum?

From Szeliski, *Computer Vision: Algorithms and Applications*

Camera matrix calibration

- Advantages:
  - very simple to formulate and solve
  - can recover K [R | t] from M using QR decomposition [Golub & VanLoan 96]
- Disadvantages:
  - doesn't compute internal parameters
  - more unknowns than true degrees of freedom
  - need a separate camera matrix for each new view

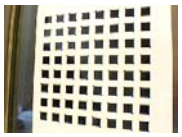
From Szeliski, *Computer Vision: Algorithms and Applications*

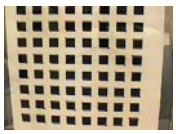
Multi-plane calibration

- Use several images of planar target held at unknown orientations [Zhang 99]
  - Compute plane homographies

$$\begin{bmatrix} u_i \\ v_i \\ 1 \end{bmatrix} \sim \mathbf{K} \begin{bmatrix} \mathbf{r}_1 & \mathbf{r}_2 & \mathbf{t} \end{bmatrix} \begin{bmatrix} x_i \\ y_i \\ 1 \end{bmatrix} \sim \mathbf{H}\mathbf{X}$$

  - Solve for K-TK-1 from Hk's
    - 1plane if only f unknown
    - 2 planes if (f,uc,vc) unknown
    - 3+ planes for full K
  - Code available from Zhang and OpenCV





From Szeliski, *Computer Vision: Algorithms and Applications*

Rotational motion

- Use pure rotation (large scene) to estimate f
  - estimate f from pairwise homographies
  - re-estimate f from 360º "gap"
  - optimize over all {K,Rj} parameters
    [Stein 95; Hartley '97; Shum & Szeliski '00; Kang & Weiss '99]

- Most accurate way to get f, short of surveying distant points

From Szeliski, *Computer Vision: Algorithms and Applications*

SFM: Structure from Motion (& Cool Robotics Share (this week))

Structure [from] Motion

- Given a set of feature tracks,
  estimate the 3D structure and 3D (camera) motion.
- Assumption: orthographic projection
- Tracks: (u_{fp},v_{fp}), f: frame, p: point
- Subtract out **mean** 2D position…

  $\mathbf{i}_f$: rotation, $\mathbf{s}_p$: position

$$u_{fp} = i_f^T s_p, v_{fp} = j_f^T s_p$$

From Szeliski, *Computer Vision: Algorithms and Applications*

Structure from motion

- How many points do we need to match?
- 2 frames:
  - (R,t): 5 dof + 3n point locations ≤
  - 4n point measurements ⇒
  - n ≥ 5

$$\hat{u}_{ij} = f(\mathbf{K}, \mathbf{R}_j, \mathbf{t}_j, \mathbf{x}_i)$$
$$\hat{v}_{ij} = g(\mathbf{K}, \mathbf{R}_j, \mathbf{t}_j, \mathbf{x}_i)$$

- k frames:
  - $6(k-1)-1 + 3n \leq 2kn$
- always want to use many more

From Szeliski, *Computer Vision: Algorithms and Applications*

Measurement equations

- Measurement equations

$u_{fp} = \boldsymbol{i}_f^T \boldsymbol{s}_p$ $\quad$ $\boldsymbol{i}_f$: rotation, $\boldsymbol{s}_p$: position

$v_{fp} = \boldsymbol{j}_f^T \boldsymbol{s}_p$

- Stack them up…

$\boldsymbol{W} = \boldsymbol{R}\,\boldsymbol{S}$

$\boldsymbol{R} = (\boldsymbol{i}_1,\ldots,\boldsymbol{i}_F,\boldsymbol{j}_1,\ldots,\boldsymbol{j}_F)^T$

$\boldsymbol{S} = (\boldsymbol{s}_1,\ldots,\boldsymbol{s}_P)$

From Szeliski, *Computer Vision: Algorithms and Applications*

Factorization

$$W = R_{2F\times 3}\, S_{3\times P}$$

SVD

$$W = U\,\Lambda\,V \quad \Lambda \text{ must be rank 3}$$

$$W' = (U\,\Lambda^{1/2})(\Lambda^{1/2}\,V) \qquad = U'\,V'$$

Make R orthogonal

$$R = QU', \quad S = Q^{-1}V'$$

$$i_f^T Q^T Q i_f = 1 \ \ldots$$

From Szeliski, *Computer Vision: Algorithms and Applications*

Results

- Look at paper figures…

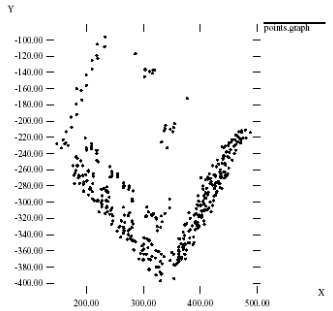


Figure 4.5: A view of the computed shape from approximately above the building (compare with figure 4.6).

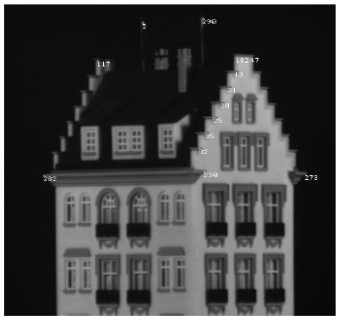


Figure 4.7: For a quantitative evaluation, distances between the features shown in the picture were measured on the actual model, and compared with the computed results. The comparison is shown in figure 4.8.

From Szeliski, *Computer Vision: Algorithms and Applications*

Bundle Adjustment

- What makes this non-linear minimization hard?
  - many more parameters: potentially slow
  - poorer conditioning (high correlation)
  - potentially lots of outliers
  - gauge (coordinate) freedom

$$\begin{aligned}\hat{u}_{ij} &= f(\mathbf{K}, \mathbf{R}_j, \mathbf{t}_j, \mathbf{x}_i) \\ \hat{v}_{ij} &= g(\mathbf{K}, \mathbf{R}_j, \mathbf{t}_j, \mathbf{x}_i)\end{aligned}$$

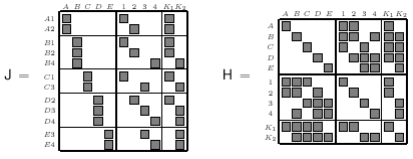
From Szeliski, *Computer Vision: Algorithms and Applications*

Lots of parameters: sparsity

- Only a few entries in Jacobian are non-zero

$$\begin{aligned}\hat{u}_{ij} &= f(\mathbf{K}, \mathbf{R}_j, \mathbf{t}_j, \mathbf{x}_i) \\ \hat{v}_{ij} &= g(\mathbf{K}, \mathbf{R}_j, \mathbf{t}_j, \mathbf{x}_i)\end{aligned}$$

$$\frac{\partial \hat{u}_{ij}}{\partial \mathbf{K}}, \quad \frac{\partial \hat{u}_{ij}}{\partial \mathbf{R}_j}, \quad \frac{\partial \hat{u}_{ij}}{\partial \mathbf{t}_j}, \quad \frac{\partial \hat{u}_{ij}}{\partial \mathbf{x}_i},$$



From Szeliski, *Computer Vision: Algorithms and Applications*

Cool Robotics Share!
(SFM & “Magic” MS Features run amok)