

Week	Date	Lecture (W: 11:10-12:40, 24-402)				
1	30-Jul	Introduction				
2	6-Aug	Representing Position & Orientation & State (Frames, Transformation Matrices & Affine Transformations)				
3	13-Aug	Robot Kinematics (& Ekka Day)				
4	20-Aug	Robot Dynamics & Control				
5	27-Aug	Robot Motion				
6	3-Sep	Robot Sensing: Perception & Multiple View Geometry				
7	10-Sep	Robot Sensing: Features & Detection using Computer Vision				
8	17-Sep	Navigation (+ Prof. M. Srinivasan)				
9	24-Sep	Localization				
	1-Oct	Study break				
10	8-Oct	State-Space Modelling				
11	15-Oct	Motion Planning + Control				
12	22-Oct	Shaping the Dynamic Response				
13	29-Oct	Linear Observers & LOR + Course Review				





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### Let's Generalize This

- Shaping the Dynamic Response
  - A method of designing a control system for a process in which all the state variables are accessible for
  - measurement-the method known as pole-placement

### • Theory:

- We will find that in a controllable system, with all the state variables accessible for measurement, it is possible to place the closed-loop poles anywhere we wish in the complex *s* plane!
- Practice:
  - Unfortunately, however, what can be attained in principle may not be attainable in practice. Speeding the response of a sluggish system requires the use of large control signals which the actuator (or power supply) may not be capable of delivering. And, control system gains are very sensitive to the location of the open-loop poles

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### **Regulator Design**

- Here the problem is to determine the gain matrix G in a linear feedback law u = -Gx G<sub>0</sub>x<sub>0</sub>
  - Where:  $x_0$  is the vector of exogenous variables. The reason it is necessary to separate the exogenous variables from the process state x, rather than deal directly with the metastate  $x = \begin{bmatrix} x \\ x_0 \end{bmatrix}$ is that we must assume that the underlying process is controllable.
    - Since the exogenous variables are not true state variables, but additional inputs that cannot be affected by the control action, they cannot be included in the state vector when using a design method that requires controllability.
    - HOWEVER, they can be used in a process for Observability!
       when we are doing this as part of the sensing/mapping process!!

• The assumption that all the state variables are accessible to measurement in the regulator means that the gain matrix G in is permitted to be any function of the state **x** that the design method requires

y = Cx $u = -G_y y$  $u = -G\hat{x}$ 

– Where:  $\hat{x}$  is the state of an appropriate dynamic system known as an "observer."

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### SISO Regulator Design [2]

• One way of determining the gains would be to set up the characteristic polynomial for *Ac*:

 $|sI - A_c| = |sI - A + bg'| = s^k + \bar{a}_1 s^{k-1} + \dots + \bar{a}_k$ 

• The coefficients  $a_1, a_2, ..., a_k$  of the powers of *s* in the characteristic polynomial will be functions of the *k* unknown gains. Equating these functions to the numerical values desired for  $a_1, a_2, ..., a_k$  will result in *k* simultaneous equations the solution of which will yield the desired gains  $g_1, ..., g_k$ .

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SISO Regulator Design [4]						
• But how to get this in companion form?						
$ar{x} = Tx$	(6.14)					
Then, as shown in Chap. 3,						
$\dot{x} = \bar{A}\bar{x} + \bar{b}u$	(6.15)					
where $\bar{A} = TAT^{-1}$ and $\bar{b} = Tb$						
For the transformed system the gain matrix is						
$ar{g}=ar{a}-ar{a}=ar{a}-a$	(6.16)					
since $\bar{a} = a$ (the characteristic equation being invariant under a variables). The desired control law in the original system is	change of state					
$u = -g'x = -g'T^{-1}\bar{x} = -\bar{g}'\bar{x}$	(6.17)					
From (6.17) we see that $\bar{g}' = g' T^{-1}$						
Thus the gain in the original system is						
$g=T'ar{g}=T'(ar{a}-a)$	(6.18)					
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# SISO Regulator Design [5]

In words, the desired gain matrix for a general system is the difference between the coefficient vectors of the desired and actual characteristic equation, premultiplied by the inverse of the transpose of the matrix T that transforms the general system into the companion form of (3.90), the A matrix of which has the form (6.11).

The desired matrix T is obtained as the product of two matrices U and V:

$$T = VU \tag{6.19}$$

The first of these matrices transforms the original system into an intermediate system

$$\dot{\tilde{x}} = \tilde{A}\tilde{x} \tag{6.20}$$

in the second companion form (3.107) and the second transformation U transforms the intermediate system into the first companion form.

Consider the intermediate system

$$\dot{\tilde{x}} = \tilde{A}\tilde{x} + \tilde{b}u \tag{6.21}$$

(6.22)

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with  $\tilde{A}$  and  $\tilde{b}$  in the form of (3.107). Then we must have

$$\tilde{A} = UAU^{-1}$$
 and  $\tilde{b} = Ub$ 

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### SISO Regulator Design [6] The desired matrix U is precisely the inverse of the controllability test matrix Q of Sec. 5.4. To prove this fact, we must show that $U^{-1}\tilde{A} = AU^{-1}$ (6.23)or $O\tilde{A} = AQ$ (6.24)Now, for a single-input system $Q = [b, Ab, \dots, A^{k-1}b]$ Thus, with $\tilde{A}$ given by (3.107), the left-hand side of (6.23) is $0 \quad 0 \quad \cdots \quad -a_k$ $Q\tilde{A} = [b, Ab, \dots, A^{k-1}b] \begin{bmatrix} 0 & 0 & \cdots & -a_{k-1} \\ 1 & 0 & \cdots & -a_{k-1} \\ 0 & 1 & \cdots & -a_{k-2} \\ \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & \cdots & -a_1 \end{bmatrix}$ = $[Ab, A^2b, \ldots, A^{k-1}b, -a_kb - a_{k-1}Ab - \cdots - a_kA^{k-1}b]$ (6.25)The last term in (6.25) is $(-a_kI - a_{k-1}A - \cdots - a_kA^{k-1})b$ (6.26)

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SISO Regulator Design [8] Multiply (6.28) by Q to obtain  $Q\tilde{b} = [b, Ab, \dots, A^{k-1}b] \begin{bmatrix} 1\\0\\\vdots\\0 \end{bmatrix} = b$ which is the same as (6.27), since  $Q^{-1} = U$ . The final step is to find the matrix V that transforms the intermediate system (6.21) into the final system (6.15). We must have  $\bar{x} = V\tilde{x}$ (6.29)For the transformation (6.28) to hold, we must have  $\bar{A} = V \tilde{A} V^{-1}$ or  $V^{-1}\vec{A} = \vec{A}V^{-1}$ (6.30)X METR 4202: Robotics 29 October 2014 - 16





# SISO Regulator Design [11]

Thus  $\tilde{b}$  and  $\bar{b}$  are the same.

The result of this calculation is that the transformation matrix T whose transpose is needed in (6.18) is the inverse of the product of the controllability test matrix and the triangular matrix (6.31).

The above results may be summarized as follows. The desired gain matrix g, by (6.18) and (6.19), is given by

$$g = (VU)'(\hat{a} - a)$$
 (6.33)

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where

 $V = W^{-1}$  and  $U = Q^{-t}$ 

Thus

$$VU = W^{-1}Q^{-1} = (QW)^{-1}$$

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LQR



# <text><text><text><text><equation-block><text>

### Optimal Regulation

The LQR problem is defined as follows. Find the control input u(t),  $t \in [0, \infty)$  that makes the following criterion as small as possible:

$$J_{\text{LQR}} := \int_0^\infty \|z(t)\|^2 + \rho \, \|u(t)\|^2 dt, \qquad (20.1)$$

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where  $\rho$  is a positive constant. The term

$$\int_0^\infty \|z(t)\|^2 dt$$

corresponds to the energy of the controlled output, and the term

$$\int_0^\infty \|u(t)\|^2 dt$$

corresponds to the *energy of the control signal*. In LQR one seeks a controller that minimizes both energies. However, decreasing the energy of the controlled output will require a large control signal, and a small control signal will lead to large controlled outputs. The role of the constant  $\rho$  is to establish a trade-off between these conflicting goals.



# **Optimal Regulation**

where  $\bar{Q} \in \mathbb{R}^{\ell \times \ell}$  and  $\bar{R} \in \mathbb{R}^{m \times m}$  are symmetric positive-definite matrices and  $\rho$  is a positive constant.

We shall consider the most general form for a quadratic criterion, which is

$$J_{\text{LQR}} := \int_0^\infty x(t)' Q x(t) + u(t)' R u(t) + 2x(t)' N u(t) dt.$$
 (J-LQR)

Since z = Gx + Hu, the criterion in (20.1) is a special form of the criterion (J-LQR) with

$$Q = G'G,$$
  $R = H'H + \rho I,$   $N = G'H$ 

and (20.2) is a special form of the criterion (J-LQR) with

$$Q = G'\bar{Q}G,$$
  $R = H'\bar{Q}H + \rho\bar{R},$   $N = G'\bar{Q}H.$ 

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# Optimal State Feedback

It turns out that the LQR criterion

$$J_{LQR} := \int_0^\infty x(t)' Q x(t) + u(t)' R u(t) + 2x(t)' N u(t) dt$$
 (J-LQR)

can be expressed as in (20.3) for an appropriate choice of feedback invariant. In fact, the feedback invariant in Proposition 20.1 will work, provided that we choose the matrix P appropriately. To check that this is so, we add and subtract this feedback invariant to the LQR criterion and conclude that

$$J_{LQR} := \int_{0}^{\infty} x'Qx + u'Ru + 2x'Nu \, dt$$
  
=  $H(x(\cdot); u(\cdot))$   
+  $\int_{0}^{\infty} x'Qx + u'Ru + 2x'Nu + (Ax + Bu)'Px + x'P(Ax + Bu) \, dt$   
=  $H(x(\cdot); u(\cdot)) + \int_{0}^{\infty} x'(A'P + PA + Q)x + u'Ru + 2u'(B'P + N')x \, dt.$ 

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# **Optimal State Feedback**

By completing the square, we can group the quadratic term in u with the cross-term in u times x:

$$(u' + x'K')R(u + Kx) = u'Ru + x'(PB + N)R^{-1}(B'P + N')x + 2u'(B'P + N')x,$$

where

$$K := R^{-1}(B'P + N'),$$

from which we conclude that

$$J_{LQR} = H(x(\cdot); u(\cdot)) + \int_0^\infty x' (A'P + PA + Q - (PB + N)R^{-1}(B'P + N'))x + (u' + x'K')R(u + Kx) dt.$$

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### **Optimal State Feedback** If we are able to select the matrix P so that $A'P + PA + Q - (PB + N)R^{-1}(B'P + N') = 0,$ (20.5)we obtain precisely an expression such as (20.3) with $\Lambda(x, u) := (u' + x'K')R(u + Kx),$ which has a minimum equal to zero for u = -Kx, $K := R^{-1}(B'P + N'),$ leading to the closed-loop system $\dot{x} = \left(A - BR^{-1}(B'P + N')\right)x.$ The following has been proved. Theorem 20.1. Assume that there exists a symmetric solution P to the algebraic Riccati equation (20.5) for which $A - BR^{-1}(B'P + N')$ is a stability matrix. Then the feedback law $u(t) := -Kx(t), \quad \forall t \ge 0,$ $K := R^{-1}(B'P + N')$ (20.6)minimizes the LQR criterion (J-LQR) and leads to $J_{LQR} := \int_0^\infty x' Qx + u' Ru + 2x' Nu \ dt = x'(0) Px(0).$ X METR 4202: Robotics 29 October 2014 - 28

### LQR In MATLAB

MATLAB<sup>(B)</sup> Hint 42 (lgr). The command [K, P, E] = lgr (A, B, Q, R, N) solves the algebraic Riccati equation

 $A'P + PA + Q - (PB + N)R^{-1}(B'P + N') = 0$ 

and computes the (negative feedback) optimal state feedback matrix gain

 $\mathbf{K} = \mathbf{R}^{-1}(\mathbf{B}'\mathbf{P} + \mathbf{N}')$ 

that minimizes the LQR criteria

$$J := \int_0^\infty x' Q x + u' R u + 2x' N u \, dt$$

for the continuous-time process

$$\dot{x} = Ax + Bu$$
.

This command also returns the poles E of the closed-loop system

 $\dot{x} = (A - BK)x.$ 

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# Case Study





















	Humanitarian	Military					
Detection	>98%	50-60%					
<b>Rate</b> (m <sup>2</sup> /day/person)	$\sim 200$	$\sim 10,000$					
Conditions	Fair weather/daytime	All weather/24/7 Army Field Manuals					
Standards	Int. Mine Action Std.						
Funding (source)	Gov't, NGOs	Military					
<ul> <li>Breeching: Line</li> <li>Demining: Area</li> <li>International Mine Action Standards (IMAS)</li> </ul>							
<ul> <li>Breeching: Line</li> <li>Demining: Area</li> <li>→ International M</li> </ul>	Jine Action Standar	rds (IMAS)					































































	Detector Imaging											
• Targets		Image: With the second secon		eent	• Designed and the second seco							
	Target#	Target type	Depth [cm]	MD	GPR							
	1	PMA-2	5	Yes	No	GPR imaging, layer 52						
	2	PMA-1A	12.5	Yes	Yes							
	3	PMA-1A	12.5	Yes	Yes	5						
	4	PMA-1A	12.5	Yes	Yes							
	5	Fragment	5	Yes	No							
	6	Stone	~10	No	Yes	GIP3, imaging; layer 55						
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