MISC: Motion Planning & Control + Shaping the Dynamic Response

METR 4202: Advanced Control & **Robotics**

Dr Surya Singh -- Lecture # 11

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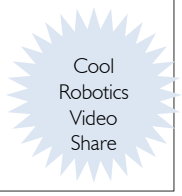
Schedule

Week	Date	Lecture (W: 11:10-12:40, 24-402)
1	30-Jul	Introduction
2	6-Aug	Representing Position & Orientation & State (Frames, Transformation Matrices & Affine Transformations)
3	13-Aug	Robot Kinematics (& Ekka Day)
4	20-Aug	Robot Dynamics & Control
5	27-Aug	Robot Motion
6	3-Sep	Robot Sensing: Perception & Multiple View Geometry
7	10-Sep	Robot Sensing: Features & Detection using Computer Vision
8	17-Sep	Navigation (+ Prof. M. Srinivasan)
9	24-Sep	Localization & Motion Planning + Control
	1-Oct	*Study break*
10	8-Oct	**State-Space Modelling**
11	15-Oct	Shaping the Dynamic Response
12	22-Oct	Linear Observers & LQR
13	29-Oct	Applications in Industry & Course Review

Announcements: Lab 2 Extended

- **Feedback from Lab 1!**
- **Lab 2 Report:**
  - Same as last lab
  - This time it's public
  - "Private" (PAF) comments are in the **online Lab2 PAF**
- **Lab 3:**
  - Cup stack/unstacking
  - Hope to get it written by this afternoon.
  - Might be a "live document" like last year
- Cool Robotics Share Site
  ➔ http://metr4202.tumblr.com/



State-Space Modelling II

State Space Definition

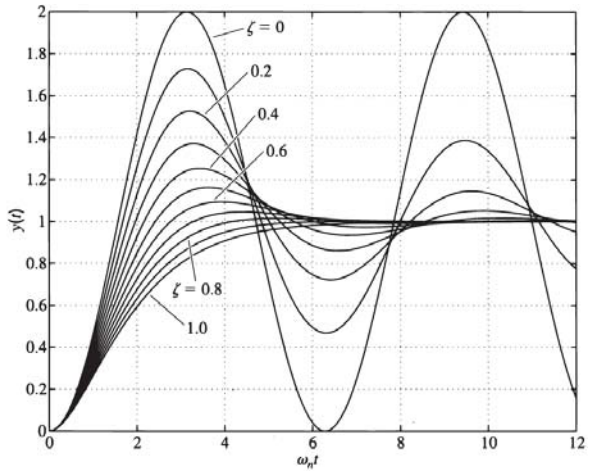
State:

- The state of a dynamic system is a set of physical quantities, the specification of which (in the absence of external excitation) completely determines the evolution of the system.
  - *specific* quantities that define the system state are not unique
  - their number (called the system *order)* is unique
  - ∴ obvious choice of the variables *(state variables)* to define the system state, but there are also many cases in which the choice of state variables is by no means obvious

2nd Order System Response

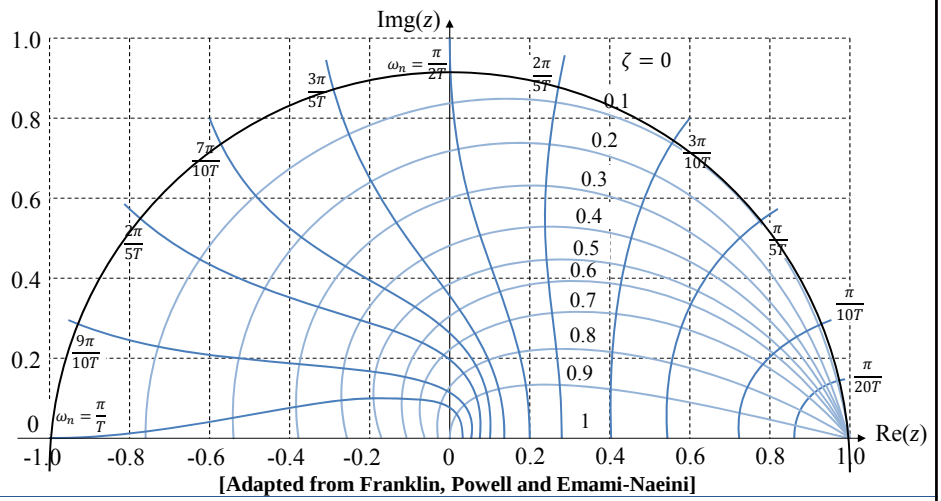
2nd Order System Response

- Response of a 2nd order system to increasing levels of damping



Damping and natural frequency

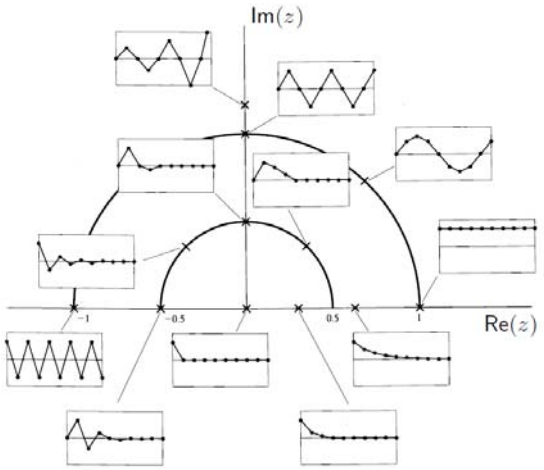
$$z = e^{sT} \text{ where } s = -\zeta\omega_n \pm j\omega_n\sqrt{1-\zeta^2}$$



[Adapted from Franklin, Powell and Emami-Naeini]

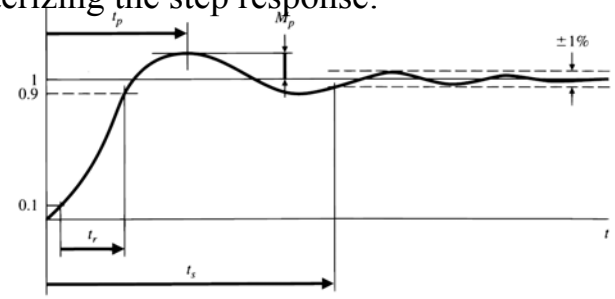
Pole positions in the z-plane

- Poles inside the unit circle are **stable**
- Poles outside the unit circle **unstable**
- Poles on the unit circle are oscillatory
- Real poles at 0 < z < 1 give exponential response
- Higher frequency of oscillation for larger
- Lower apparent damping for larer and r



2nd Order System Specifications

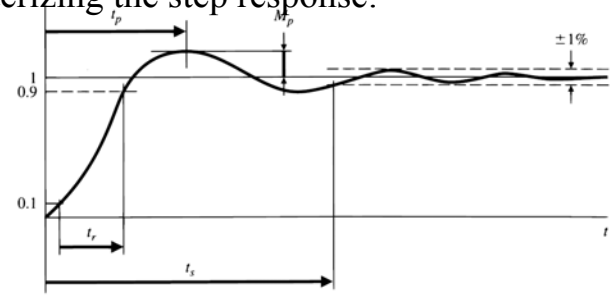
Characterizing the step response:



- Rise time (10% → 90%): $t_r \approx \frac{1.8}{\omega_0}$
- Overshoot: $M_p \approx \frac{e^{-\pi\zeta}}{\sqrt{1-\zeta^2}}$
- Settling time (to 1%): $t_s = \frac{4.6}{\zeta\omega_0}$
- Steady state error to unit step: e_{ss}
- Phase margin: $\phi_{PM} \approx 100\zeta$

2nd Order System Specifications

Characterizing the step response:



- Rise time (10% → 90%) & Overshoot:
  t_r, M_p → ζ, ω_0 : Locations of dominant poles
- Settling time (to 1%):
  t_s → radius of poles: $|z| < 0.01^{\frac{T}{t_s}}$
- Steady state error to unit step:
  e_{ss} → final value theorem $e_{ss} = \lim_{z \to 1} \{(z-1) F(z)\}$

Ex: System Specifications → Control Design [1/4]

Design a controller for a system with:

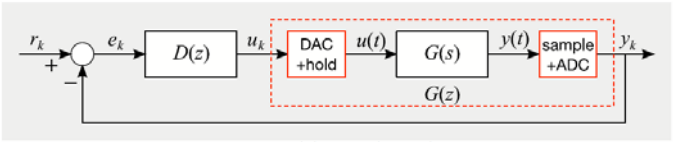
- A continuous transfer function: $G(s) = \frac{0.1}{s(s+0.1)}$
- A discrete ZOH sampler
- Sampling time (T_s): T_s= 1s
- $C \; u_k = -0.5u_{k-1} + 13\,(e_k - 0.88e_{k-1})$

The closed loop system is required to have:

- $M_p < 16\%$
- $t_s < 10$ s
- $e_{ss} < 1$

Ex: System Specifications → Control Design [2/4]

1. (a) Find the pulse transfer function of $G(s)$ plus the ZOH



$$G(z) = (1 - z^{-1})\mathcal{Z}\left\{\frac{G(s)}{s}\right\} = \frac{(z-1)}{z}\mathcal{Z}\left\{\frac{0.1}{s^2(s+0.1)}\right\}$$

e.g. look up $\mathcal{Z}\{a/s^2(s+a)\}$ in tables:

$$G(z) = \frac{(z-1)}{z}\,\frac{z\Big((0.1-1+e^{-0.1})z + (1-e^{-0.1}-0.1e^{-0.1})\Big)}{0.1(z-1)^2(z-e^{-0.1})}$$
$$= \frac{0.0484(z+0.9672)}{(z-1)(z-0.9048)}$$

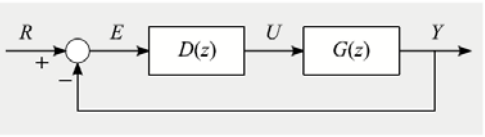
(b) Find the controller transfer function (using z = shift operator):

$$\frac{U(z)}{E(z)} = D(z) = 13\frac{(1-0.88z^{-1})}{(1+0.5z^{-1})} = 13\frac{(z-0.88)}{(z+0.5)}$$

Ex: System Specifications → Control Design [3/4]

2. Check the steady state error e_{ss} when r_k = unit ramp

$$e_{ss} = \lim_{k\to\infty} e_k = \lim_{z\to 1}(z-1)E(z)$$



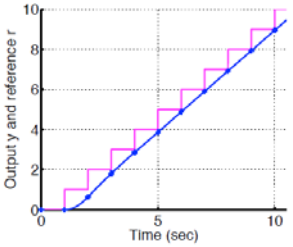
$$\frac{E(z)}{R(z)} = \frac{1}{1+D(z)G(z)}$$
$$R(z) = \frac{Tz}{(z-1)^2}$$

so $e_{ss} = \lim_{z\to 1}\left\{(z-1)\frac{Tz}{(z-1)^2}\frac{1}{1+D(z)G(z)}\right\} = \lim_{z\to 1}\frac{T}{(z-1)D(z)G(z)}$

$$= \lim_{z\to 1}\frac{T}{(z-1)\dfrac{0.0484(z+0.9672)}{(z-1)(z-0.9048)}D(1)}$$

$$= \frac{1-0.9048}{0.0484(1+0.9672)D(1)} = 0.96$$

$\Longrightarrow$ $e_{ss} < 1$ (as required)



Ex: System Specifications → Control Design [4/4]

3. Step response: overshoot $M_p < 16\% \implies \zeta > 0.5$

   settling time $t_s < 10 \implies |z| < 0.01^{1/10} = 0.63$

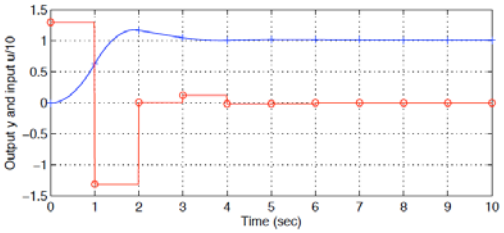
The closed loop poles are the roots of $1 + D(z)G(z) = 0$, i.e.

$$1 + 13\frac{(z-0.88)}{(z+0.5)}\frac{0.0484(z+0.9672)}{(z-1)(z-0.9048)} = 0$$

$$\implies z = 0.88,\ -0.050 \pm j0.304$$

But the pole at $z = 0.88$ is cancelled by controller zero at $z = 0.88$, and

$$z = -0.050 \pm j0.304 = re^{\pm j\theta} \implies \begin{cases} r = 0.31,\ \theta = 1.73 \\ \zeta = 0.56 \end{cases}$$



all specs satisfied!

Example:
(Back To) Robot Arms

Slides 17-27 Source: R. Lindeke, ME 4135, "Introduction to Control"

Remembering the Motion Models:

- Recall from Dynamics, the Required Joint Torque is:

$$\tau_i = D_i(q)\,\ddot{q}_i + C_i(q, \dot{q}_i) + h(q) + b(\dot{q}_i)$$

- $D_i(q)\,\ddot{q}_i$: Dynamical Manipulator Inertial Tensor – a function of position and acceleration
- $C_i(q, \dot{q}_i)$: Coupled joint effects (centrifugal and coriolis) issues due to multiple moving joints
- $h(q)$: Gravitational Effects
- $b(\dot{q}_i)$: Frictional Effect due to Joint/Link movement

Lets simplify the model

- This torque model is a 2nd order one (in position) lets look at it as a velocity model rather than positional one then it becomes a system of highly coupled 1st order differential equations

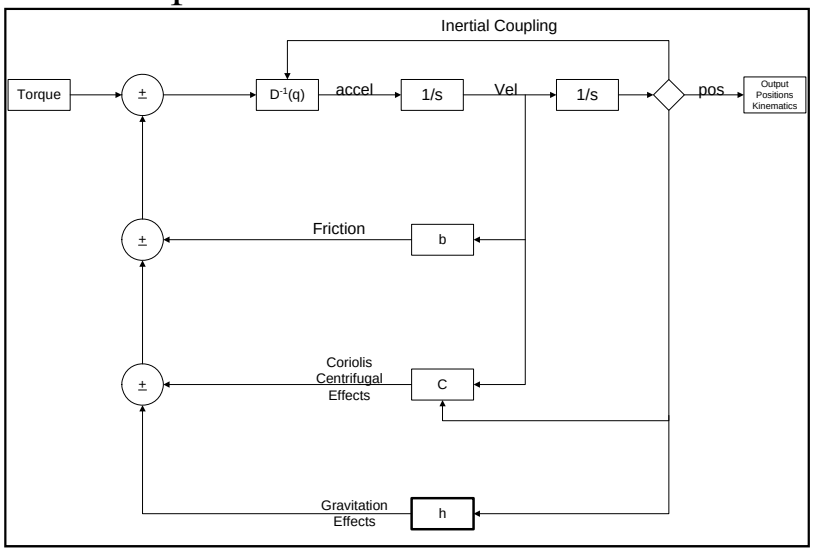
- We will then isolate Acceleration terms (acceleration is the 1st derivative of velocity)

$$a = \dot{v} = \ddot{q} = D_i^{-1}(q)\,(\tau_i - C_i(q, \dot{q}_i) - h(q) - b(\dot{q}_1))$$

Considering Control:

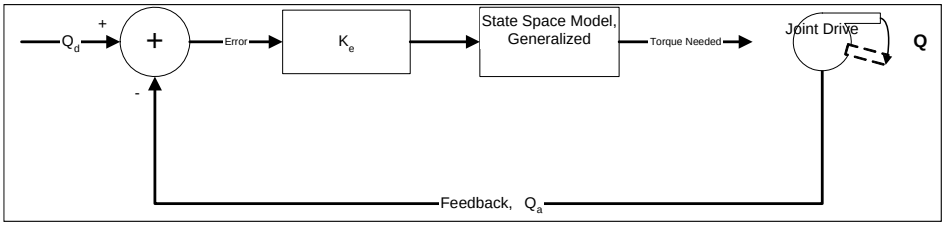
- Each Link's torque is influenced by each other links motion
  - We say that the links are highly coupled
- Solution then suggests that control should come from a simultaneous solution of these torques
- We will model the solution as a "State Space" design and try to balance the torque-in with *positional control*-out – the most common way it is done!
  - But we could also use 'force control' to solve the control problem!

The State-Space Control Model:



Setting up a Real Control

- We will (start) by using positional error to drive our torque devices



- This simple model is called a PE (proportional error) controller

PE Controller:

- To a 1st approximation, $\tau = K_m * I$
  - Torque is proportional to motor current

- And the Torque required is a function of 'Inertial' (Acceleration) and 'Friction' (velocity) effects as suggested by our L-E models

$$\tau_m \simeq J_{eq}\ddot{q} + F_{eq}\dot{q}$$

→ Which can be approximated as:

$$K_m I_m = J_{eq}\ddot{q} + F_{eq}\dot{q}$$

Setting up a "Control Law"

- We will use the positional error (as drawn in the state model) to develop our torque control
- We say then for PE control:

$$\tau \propto k_{pe}(\theta_d - \theta_a)$$

- Here, k_{pe} is a "gain" term that guarantees sufficient current will be generated to develop appropriate torque based on observed positional error

Using this Control Type:

- It is a representation of the physical system of a mass on a spring!
- We say after setting our target as a 'zero goal' that:

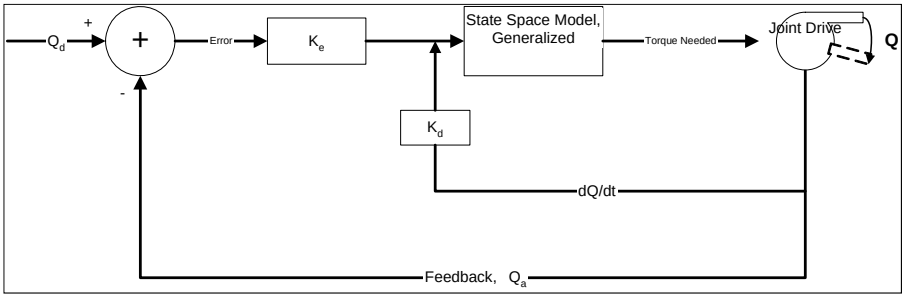
$$-k_{pe} * \theta_a = J\ddot{\theta} + F\dot{\theta}$$

the solution of which is:

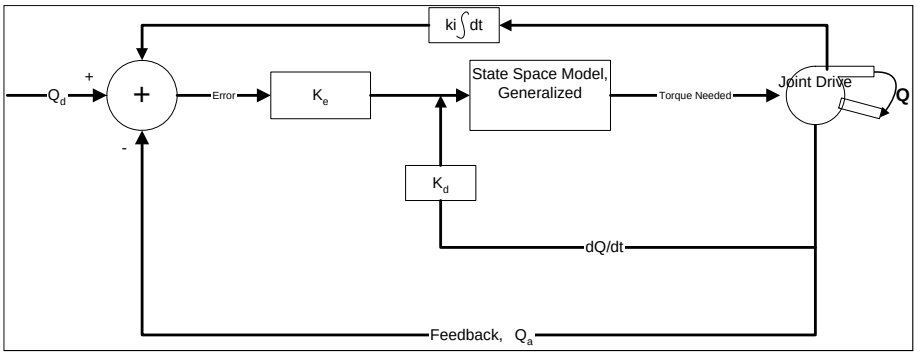
θ_a is a function of the servo feedback as a function of time!

$$\theta_a = e^{-(F/2J)t}\left[C_1 e^{(1/2)\omega t} + C_2 e^{-(1/2)\omega t}\right]$$

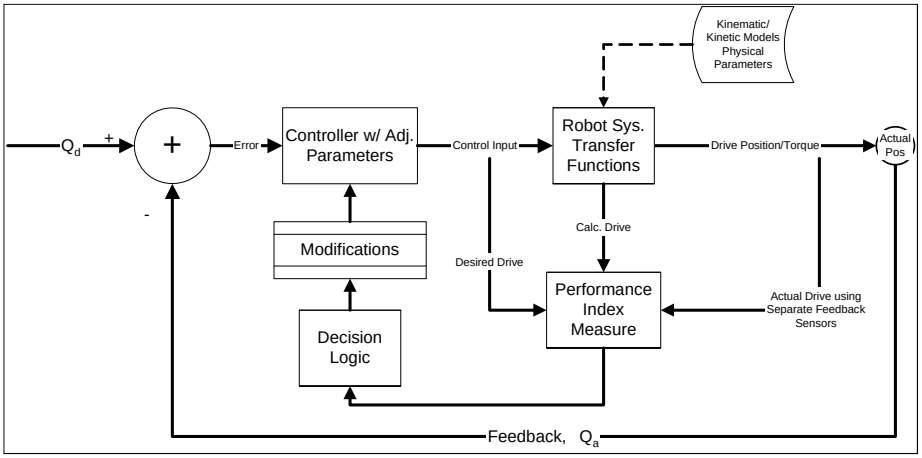
State Space Model of PD:



PID State Space Model:



State Model of Adjustable Controller



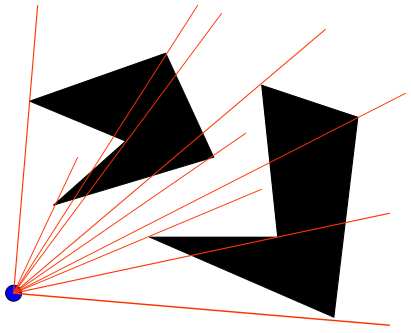
From Last Week: Motion Planning

Path-Planning Approaches

- Roadmap
  Represent the connectivity of the free space by a network of 1-D curves
- Cell decomposition
  Decompose the free space into simple cells and represent the connectivity of the free space by the adjacency graph of these cells
- Potential field
  Define a function over the free space that has a global minimum at the goal configuration and follow its steepest descent

Slide from Latombe, CS326A

I. Rotational Sweep



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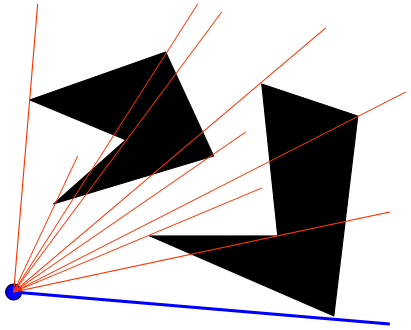
Rotational Sweep



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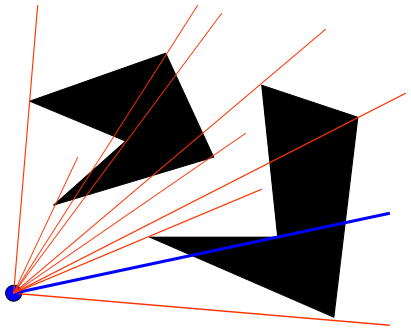
Rotational Sweep



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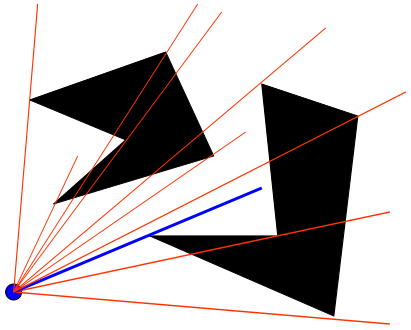

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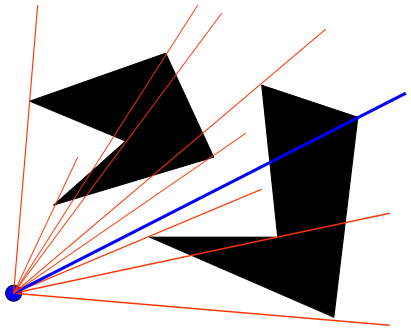

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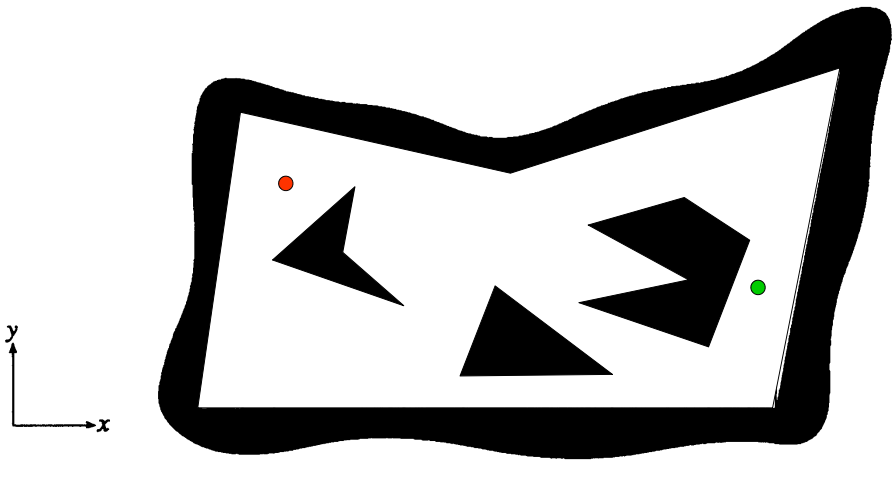
II. Cell-Decomposition Methods

Two classes of methods:

- Exact cell decomposition
  - The free space **F** is represented by a collection of non-overlapping cells whose union is exactly **F**
  - Example: trapezoidal decomposition
- Approximate cell decomposition
  - **F** is represented by a collection of non-overlapping cells whose union is contained in **F**
    Examples: quadtree, octree, 2n-tree

Trapezoidal decomposition



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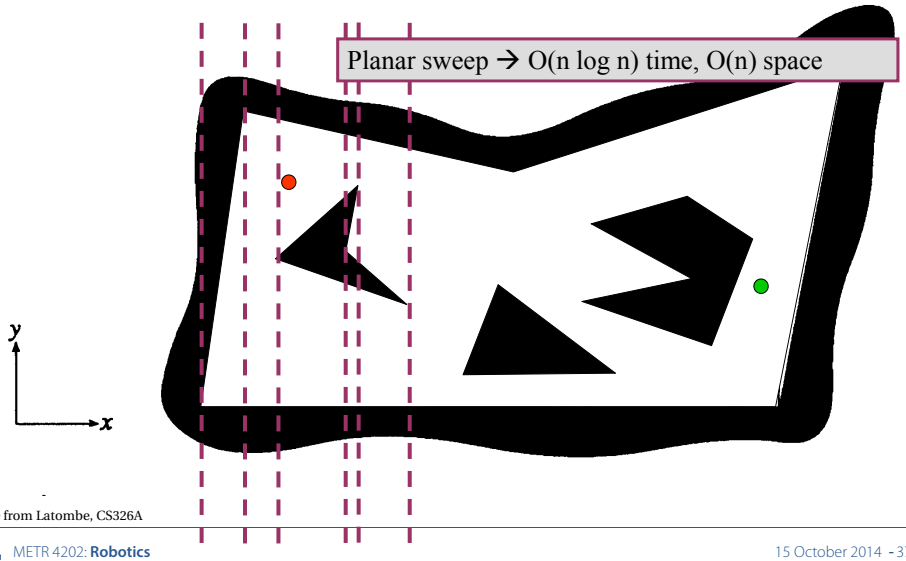
Trapezoidal decomposition



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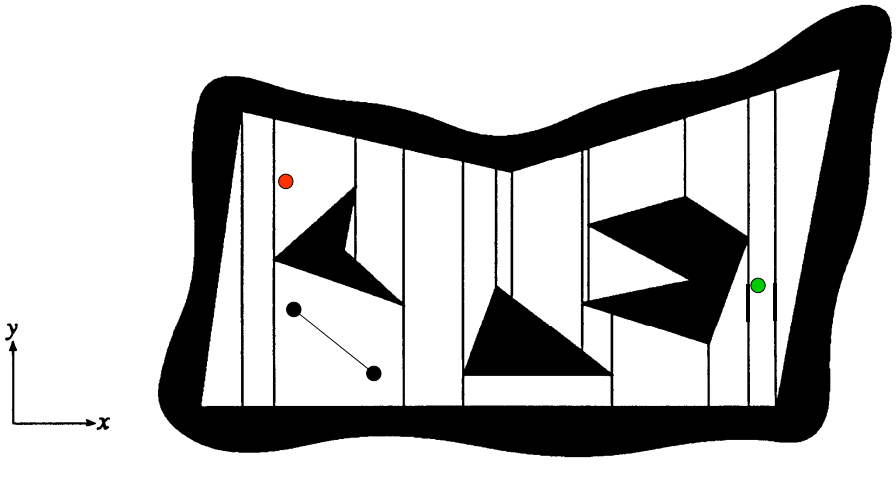
Trapezoidal decomposition



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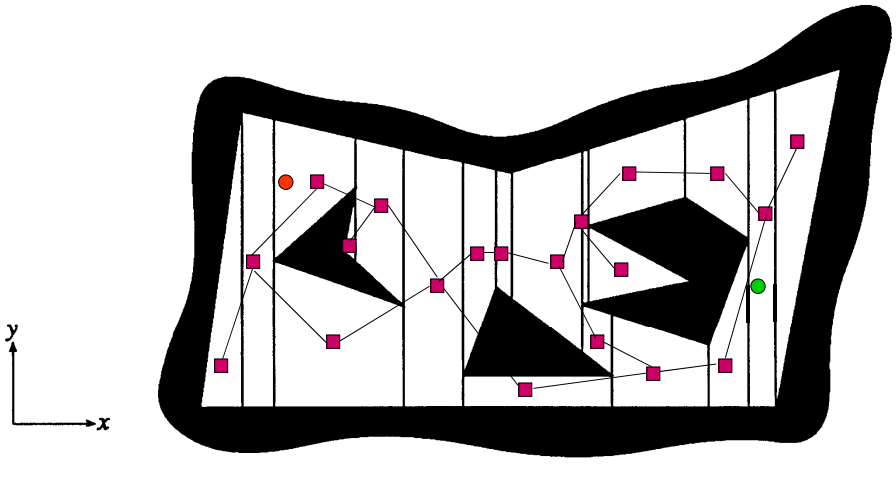
Trapezoidal decomposition



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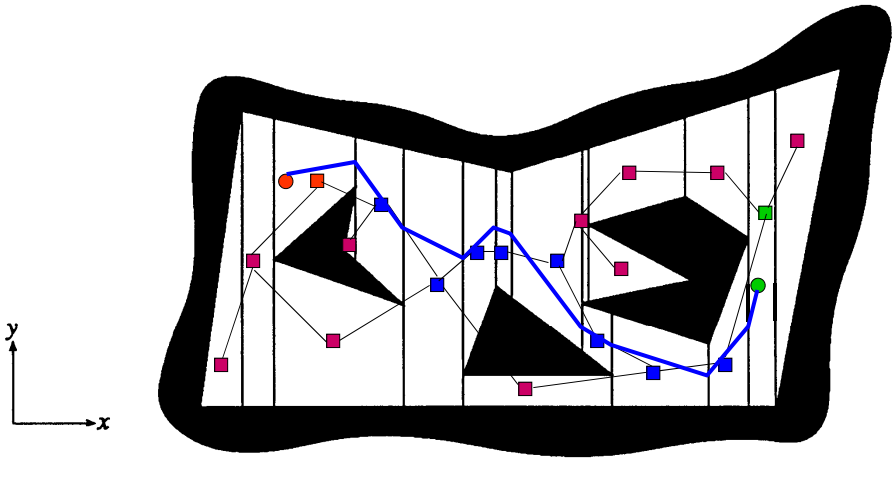
Trapezoidal decomposition



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III. Roadmap Methods

- **Visibility graph**
- **Voronoi diagram**
- Silhouette
  First complete general method that applies to spaces of any dimension and is singly exponential in # of dimensions [Canny, 87]
- **Probabilistic roadmaps (PRMS) and Rapidly-exploring Randomized Trees (RRTs)**

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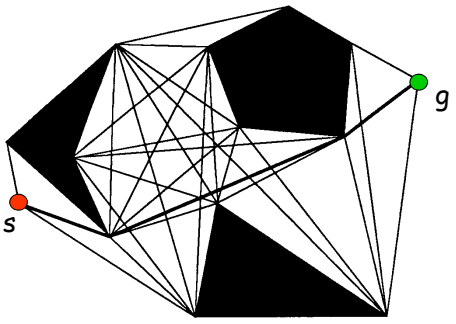
Roadmap Methods

- **Visibility graph**
  Introduced in the Shakey project at SRI in the late 60s.
  Can produce shortest paths in 2-D configuration spaces



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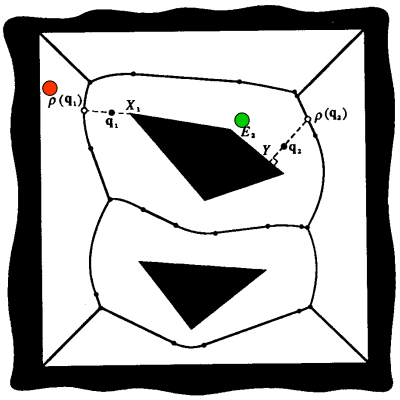
Roadmap Methods

- Voronoi diagram Introduced by Computational Geometry researchers. Generate paths that maximizes clearance.

  O(n log n) time
  O(n) space



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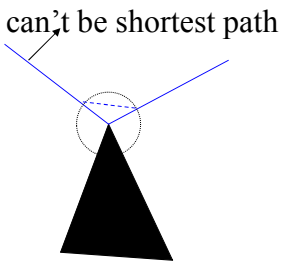
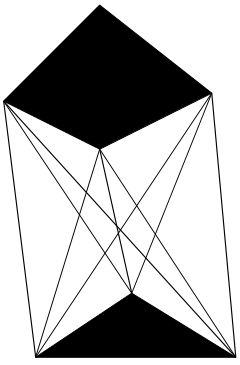
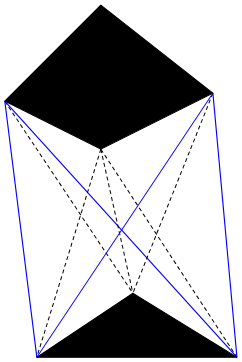
II. Visibility Graph







tangent segments

→ Eliminate concave obstacle vertices

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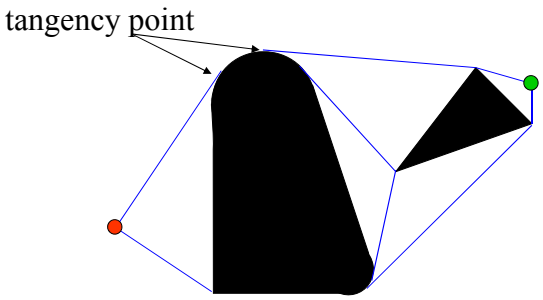
Generalized (Reduced) -- Visibility Graph



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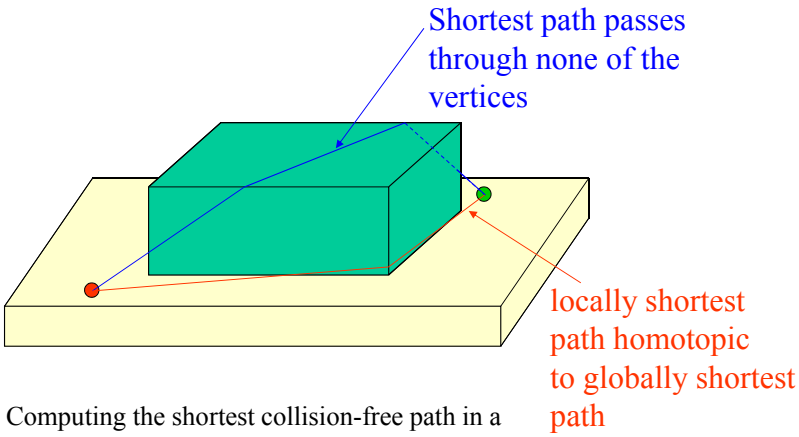
Three-Dimensional Space

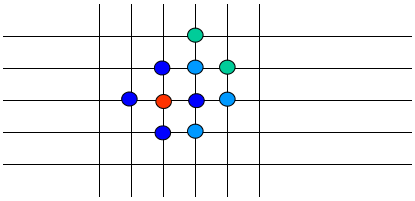


Computing the shortest collision-free path in a polyhedral space is NP-hard

Slide from Latombe, CS326A

Sketch of Grid Algorithm (with best-first search)

- Place regular grid G over space
- Search G using best-first search algorithm with potential as heuristic function



Slide from Latombe, CS326A

Simple Algorithm (for Visibility Graphs)

- Install all obstacles vertices in VG, plus the start and goal positions
- For every pair of nodes u, v in VG

```
If segment(u,v) is an obstacle edge then
        insert (u,v) into VG
else
for every obstacle edge e
        if segment(u,v) intersects e
                then go up to segment
insert (u,v) into VG
```

- Search VG using A*

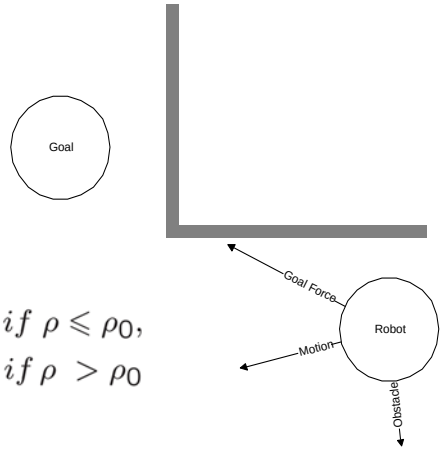
Slide based on Latombe, CS326A

IV. Potential Field Methods

- Approach initially proposed for real-time collision avoidance [Khatib, 86]

$$F_{Goal} = -k_p(x - x_{Goal})$$

$$F_{Obstacle} = \begin{cases} \eta \left(\frac{1}{\rho} - \frac{1}{\rho_0}\right) \frac{1}{\rho^2} \frac{\partial \rho}{\partial x} & if\ \rho \leqslant \rho_0, \\ 0 & if\ \rho > \rho_0 \end{cases}$$



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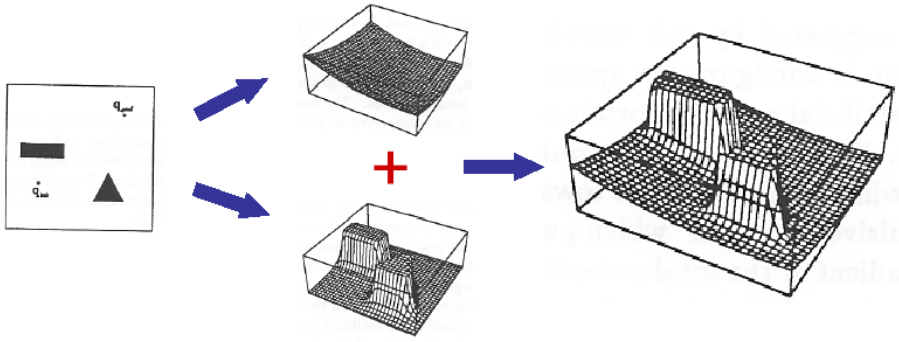
Attractive and Repulsive fields



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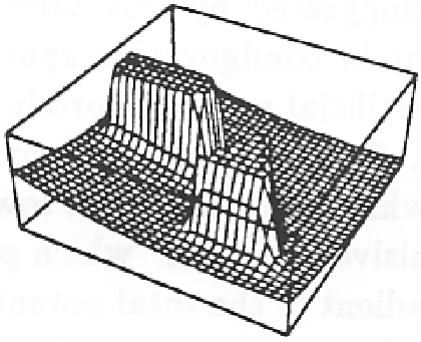
Local-Minimum Issue



- Perform best-first search (possibility of combining with approximate cell decomposition)
- Alternate descents and random walks
- Use local-minimum-free potential (navigation function)

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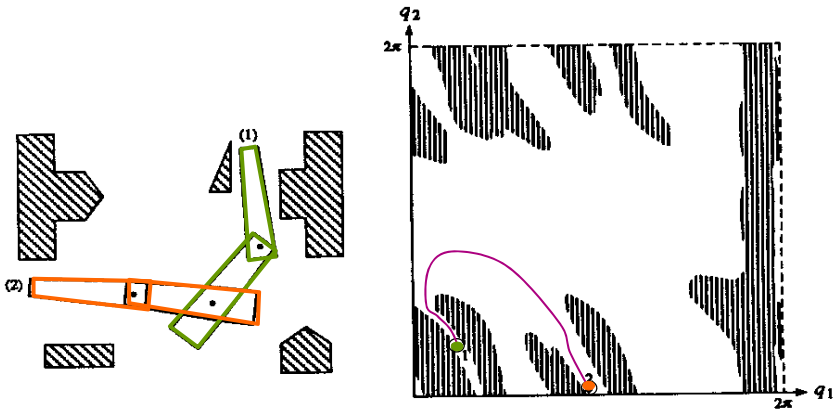
Configuration Space



- A robot configuration is a specification of the positions of all robot points relative to a fixed coordinate system
- Usually a configuration is expressed as a "vector" of position/orientation parameters

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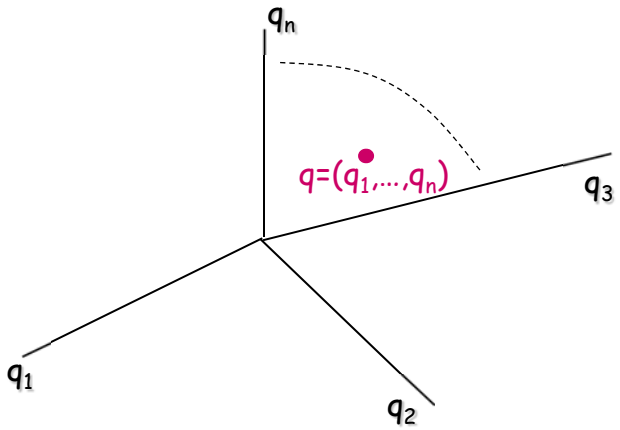
Motion Planning in C-Space



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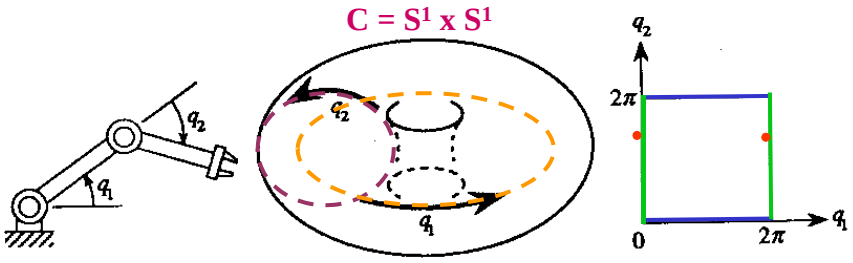
Configuration Space of a Robot

- Space of all its possible configurations
- But the topology of this space is usually not that of a Cartesian space



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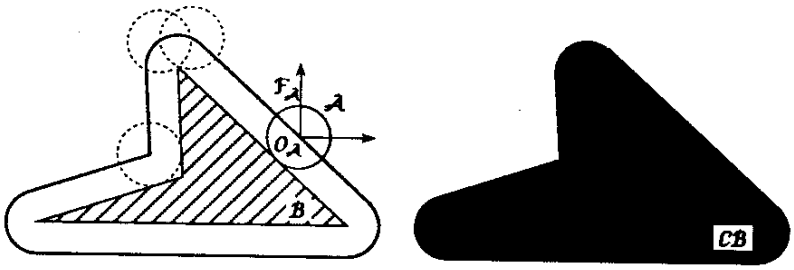
Disc Robot in 2-D Workspace



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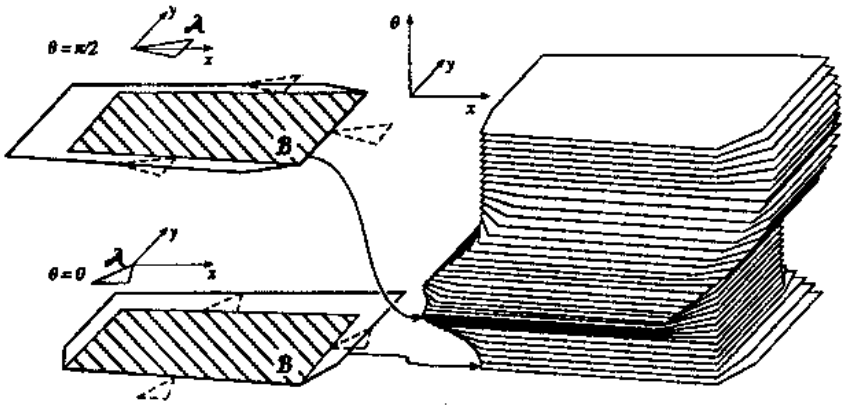
Rigid Robot Translating and Rotating in 2-D



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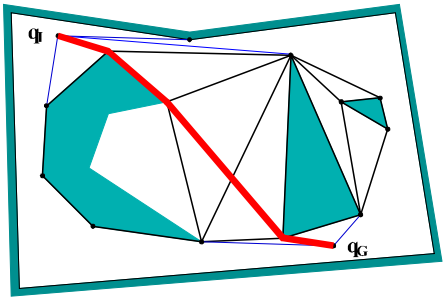
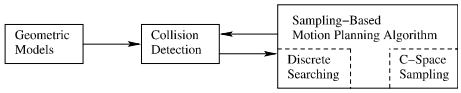
Geometric Planning Methods



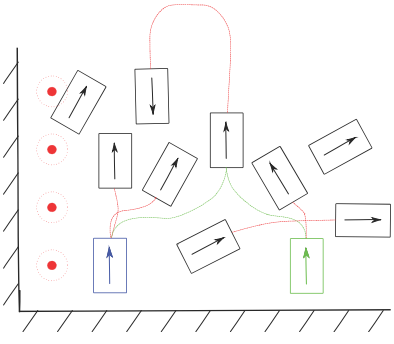
- Several Geometric Methods:
  - Vertical (Trapezoidal) Cell Decomposition
  - **Roadmap Methods**
    - **Cell (Triangular) Decomposition**
    - Visibility Graphs
    - Veroni Graphs

Artwork from LaValle, Ch. 6

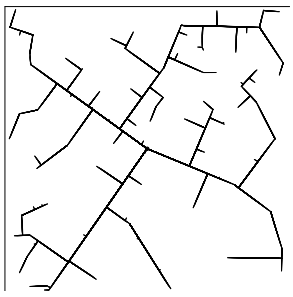
Sample-Based Motion Planning



- PRMs

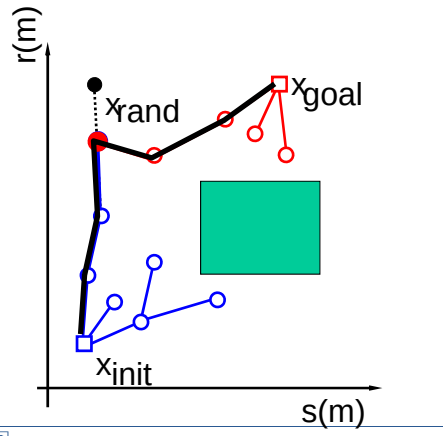


- RRTs

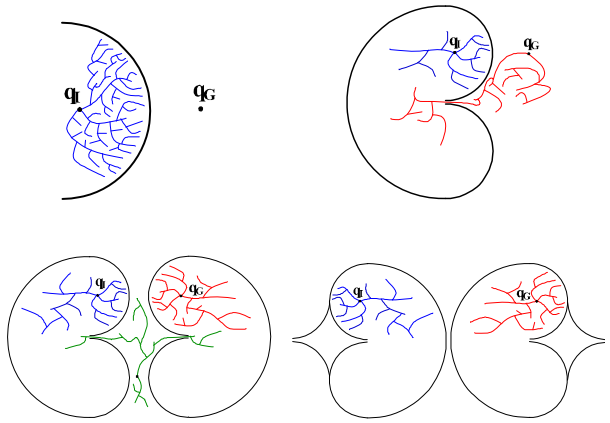


Artwork based on LaValle, Ch. 5

Rapidly Exploring Random Trees (RRT)

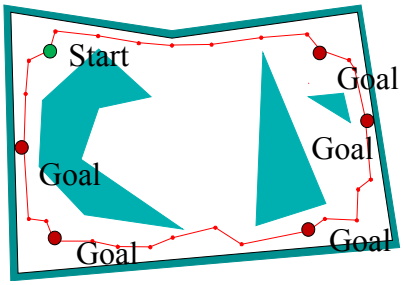


Sampling and the "Bug Trap" Problem



Artwork based on LaValle, Ch. 5

Multiple Points & Sequencing



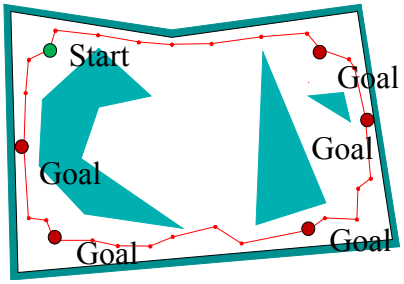
- Sequencing
  - Determining the "best" order to go in

➔ Travelling Salesman Problem

A salesman has to visit each city on a given list exactly once. In doing this, he **starts** from his home city and in the **end he has to return to his home** city. It is plausible for him to select the order in which he visits the cities so that the **total of the distances travelled** in his tour is as small as possible.

Artwork based on LaValle, Ch. 6

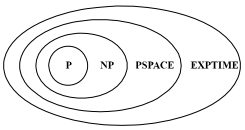
Travelling Salesman Problem



- Given a $n \times n$ distance matrix $\mathbf{C}=(c_{ij})$
- Minimize:

$$c(\pi) = \sum_{i=1}^{n} c_{i\pi(i)}$$

- Note that this problem is NP-Hard

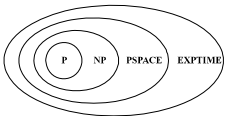


➔ BUT, Special Cases are Well-Solvable!

Artwork based on LaValle, Ch. 6

Travelling Salesman Problem [2]

- This problem is NP-Hard



➔ BUT,
Special Cases are
Well-Solvable!

For the Euclidean case
(where the points are on the 2D Euclidean plane) :

- The shortest TSP tour does not intersect itself, and thus geometry makes the problem somewhat easier.
- If all cities lie on the boundary of a convex polygon, the optimal tour is a cyclic walk along the boundary of the polygon (in clockwise or counterclockwise direction).

***The k*-line TSP**

- The a special case where the cities lie on *k* parallel (or almost parallel) lines in the Euclidean plane.
- EG: Fabrication of printed circuit boards
- Solvable in $\mathbf{O}(n^3)$ time by Dynamic Programming (Rote's algorithm)

The necklace TSP

- The special Euclidean TSP case where there exist *n* circles around the *n* cities such that every cycle intersects exactly two adjacent circles