METR4202 -- Robotics
Tutorial 4 – Week 5: Robot Dynamics and Control

Reading
Please read/review Chapters 8 and 9 of Robotics, Vision and Control.

Review: Forward Kinematics of a two-link planar manipulator

$$x = a_1 cos\theta_1 + a_2 \cos(\theta_1 + \theta_2)$$
$$y = a_1 \sin\theta_1 + a_2 \sin(\theta_1 + \theta_2)$$

Questions

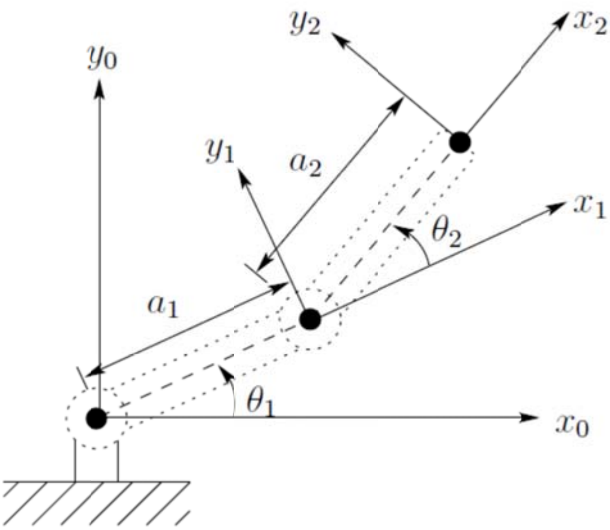

Figure 1: Two-link planar manipulator

1.

a.) Using the two-link planar manipulator from the previous tutorial ([Ekka Day]), calculate the Jacobian needed to relate the joint velocities to the tool-point velocities.

b.) Similarly, calculate the inverse Jacobian needed to relate the tool-point velocities to the joint velocities.

2.

a.) Using the Jacobian found in Q1a, calculate the tool point linear velocity if joint 1 is rotating at 1 rad/s and joint 2 is rotating at 3 rad/s.

b.) Calculate the resulting joint torques τ, given a force F = (30, -20) is applied to the end effector tool point.

3.

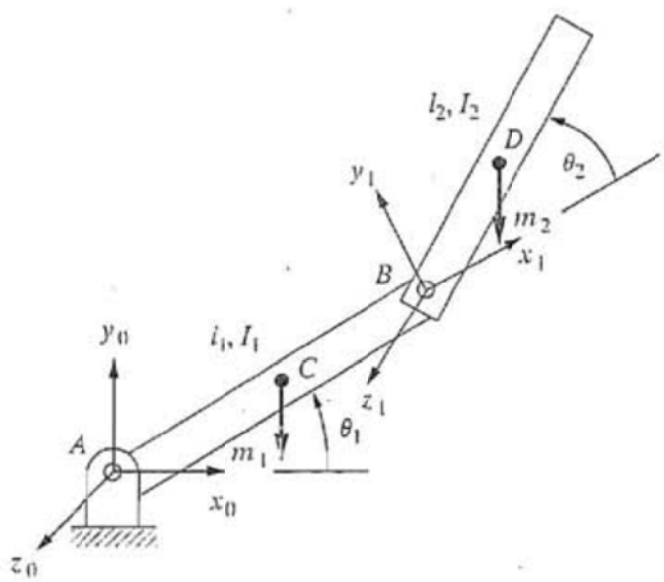

Figure 2: Two-link revolute joint arm.

a.) With respect to figure 2 above, derive the equations of motion for the two-degree-of-freedom robot arm using the Lagrangian method.

Recall:
Mass Moment of inertia of a slender rod:
Taken about the center of mass:

$$I = \frac{1}{12}ml^2$$

Taken about a fixed axis of rotation:

$$I = \frac{1}{3}ml^2$$

Potential Energy = mgh

Challenge Question:

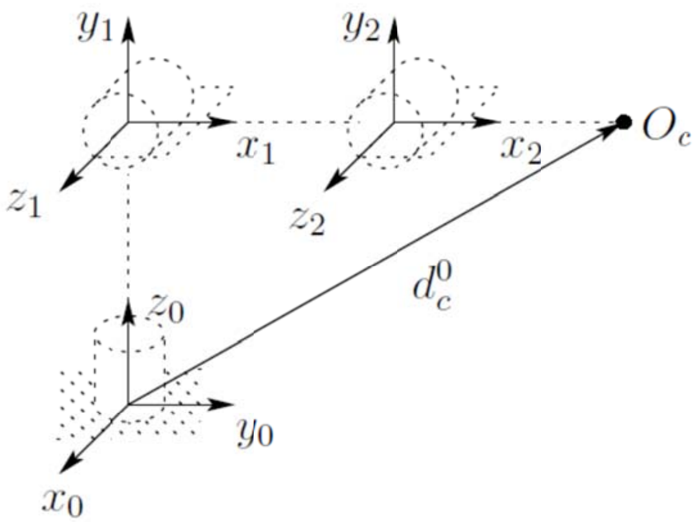

Figure 3: Elbow Manipulator

a.) List the DH parameters for this arm, clearly indication which parameters are the joint variables.

b.) Find the inverse Kinematic equations for the arm to derive the joint values from tool point position.

c.) Given that the tool point is at $(1.0\text{m}, 0.2\text{m}, 0.5\text{m})^T$, use the inverse kinematic equations to find the joint values.

d.) Find the manipulator Jacobian, J, that relates the joint velocities to the tool point velocity.