Kinematics

"Motion of it's own will"

METR 4202: Advanced Control & **Robotics**

Dr Surya Singh

Lecture # 3

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metr4202@itee.uq.edu.au

http://itee.uq.edu.au/~metr4202/

Schedule

Week	Date	Lecture (F: 9-10:30, 42-212)
1	26-Jul	Introduction
2	2-Aug	Representing Position & Orientation & State (Frames, Transformation Matrices & Affine Transformations)
3	**9-Aug**	**Robot Kinematics**
4	16-Aug	Robot Dynamics & Control
5	23-Aug	Sensors & Measurement
6	30-Aug	Perception
7	6-Sep	Computer Vision & Localization (SFM/SLAM)
8	13-Sep	Localization and Navigation
9	20-Sep	State-Space Modelling
	27-Sep	State-Space Control
10	4-Oct	*Study break*
11	11-Oct	Motion Planning
12	18-Oct	Vision-based control (+ Prof. P. Corke or + Prof. M. Srinivasan)
13	25-Oct	Applications in Industry (+ Prof. S. LaValle) & Course Review

Announcements:

- Tutorials via Platypus
- +2% of the "Final Quiz" Extra-Credit
  for **fully** completing the tutorial and peer review ?
  (remember: you have to submit questions in order to peer review)

- Tutorials: Due Saturday Night
- Tutorial Peer Feedback: Due Tuesday Night
  [Strict – No extensions regardless!]

Cool Robotics Video Share

Cool Robotics Share

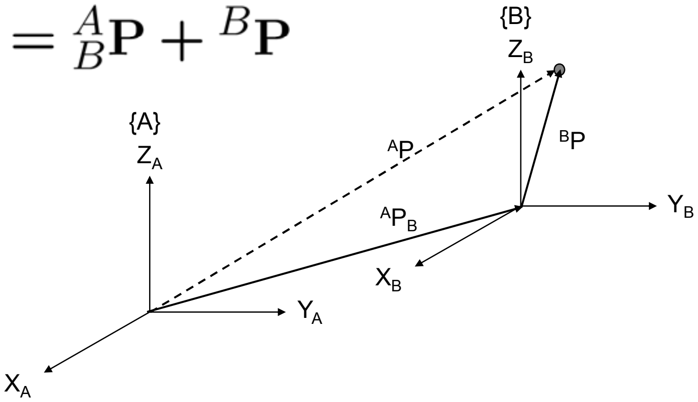
Continuing from Last Week…

Coordinate Transformations [1]

- Translation Again:

  If {B} is translated with respect to {A} **without rotation**, then it is a vector sum

$$ {}^{A}\mathbf{P} = {}^{A}_{B}\mathbf{P} + {}^{B}\mathbf{P} $$



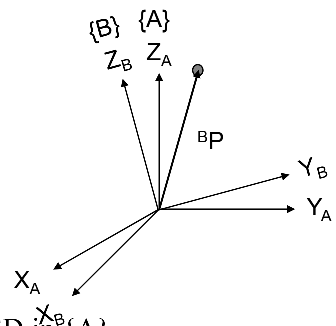
Coordinate Transformations [2]

- Rotation Again:

  {B} is rotated with respect to {A} then
  use rotation matrix to determine new components

- NOTE: $${}^{A}\mathbf{P} = {}^{A}_{B}\mathbf{R}\,{}^{B}\mathbf{P}$$
  - The Rotation matrix's ***subscript*** matches the position vector's **superscript**

$${}^{A}\mathbf{P} = {}^{A}_{[\![B]\!]}\mathbf{R}\,{}^{[\![B]\!]}\mathbf{P}$$



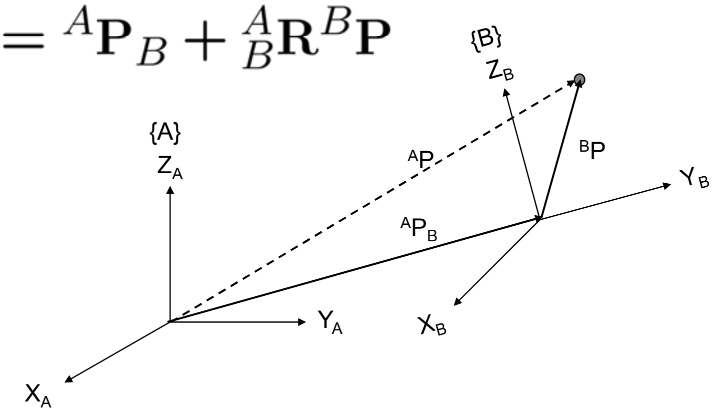
  - This gives Point Positions of {B} ORIENTED in {A}

Coordinate Transformations [3]

- Composite transformation:

  {B} is moved with respect to {A}:

$${}^{A}\mathbf{P} = {}^{A}\mathbf{P}_{B} + {}^{A}_{B}\mathbf{R}\,{}^{B}\mathbf{P}$$



General Coordinate Transformations [1]

- A compact representation of the translation and rotation is known as the **Homogeneous Transformation**

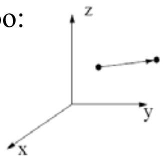
$$ {}^{A}_{B}\mathbf{T} = \begin{bmatrix} & {}^{A}_{B}\mathbf{R} & & {}^{A}\mathbf{P}_{B} \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

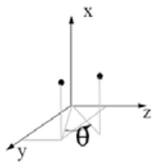
- This allows us to cast the rotation and translation of the general transform in a single matrix form

$$ \begin{bmatrix} {}^{A}\mathbf{P} \\ 1 \end{bmatrix} = {}^{A}_{B}\mathbf{T} \begin{bmatrix} {}^{B}\mathbf{P} \\ 1 \end{bmatrix} $$

General Coordinate Transformations [2]

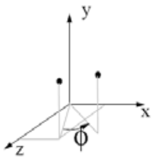
- Similarly, fundamental orthonormal transformations can be represented in this form too:

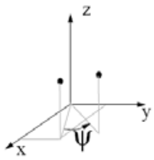




$$ Trans(u, v, w) = \begin{bmatrix} 1 & 0 & 0 & u \\ 0 & 1 & 0 & v \\ 0 & 0 & 1 & w \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

$$ Rotx(\theta) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & c\theta & -s\theta & 0 \\ 0 & s\theta & c\theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$





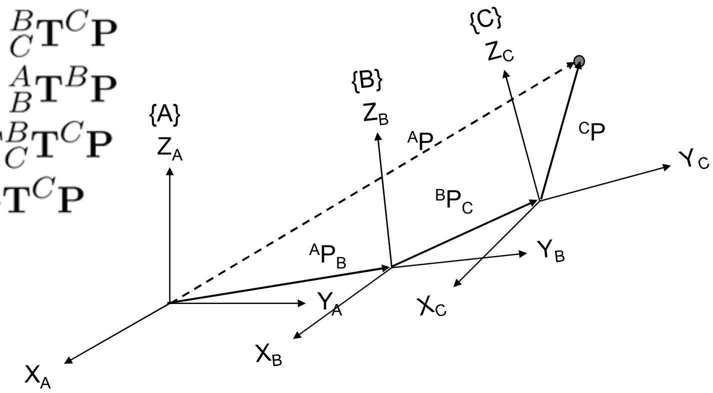
$$ Roty(\phi) = \begin{bmatrix} c\phi & 0 & s\phi & 0 \\ 0 & 1 & 0 & 0 \\ -s\phi & 0 & c\phi & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

$$ Rotz(\psi) = \begin{bmatrix} c\psi & -s\psi & 0 & 0 \\ s\psi & c\psi & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

General Coordinate Transformations [3]

- Multiple transformations compounded as a chain

$$
\begin{aligned}
{}^{B}\mathbf{P} &= {}^{B}_{C}\mathbf{T}\,{}^{C}\mathbf{P} \\
{}^{A}\mathbf{P} &= {}^{A}_{B}\mathbf{T}\,{}^{B}\mathbf{P} \\
&= {}^{A}_{B}\mathbf{T}\,{}^{B}_{C}\mathbf{T}\,{}^{C}\mathbf{P} \\
&= {}^{A}_{C}\mathbf{T}\,{}^{C}\mathbf{P}
\end{aligned}
$$



$$
{}^{A}_{C}\mathbf{T} = \begin{bmatrix} {}^{A}_{B}\mathbf{R}\,{}^{B}_{C}\mathbf{R} & {}^{A}\mathbf{P}_{B} + {}^{A}_{B}\mathbf{R}\,{}^{B}\mathbf{P}_{C} \\ 0 \quad 0 \quad 0 & 1 \end{bmatrix}
$$

Homogenous Coordinates

$$
\widehat{p} = \begin{bmatrix} \rho p_x & \rho p_y & \rho p_z & \rho \end{bmatrix}^T
$$

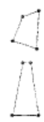
- ρ is a scaling value

Homogenous Transformation

$$\begin{bmatrix} {}^{A}R_{B} & {}^{A}p \\ \gamma & \rho \end{bmatrix}$$

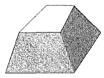
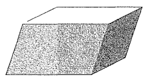
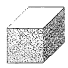
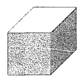
- γ is a projective transformation

Projective Transformations …

Group	Matrix	Distortion	Invariant properties
Projective 8 dof	$\begin{bmatrix} h_{11} & h_{12} & h_{13} \\ h_{21} & h_{22} & h_{23} \\ h_{31} & h_{32} & h_{33} \end{bmatrix}$		Concurrency, collinearity, **order of contact**: intersection (1 pt contact); tangency (2 pt contact); inflections (3 pt contact with line); tangent discontinuities and cusps. cross ratio (ratio of ratio of lengths).
Affine 6 dof	$\begin{bmatrix} a_{11} & a_{12} & t_x \\ a_{21} & a_{22} & t_y \\ 0 & 0 & 1 \end{bmatrix}$		Parallelism, ratio of areas, ratio of lengths on collinear or parallel lines (e.g. midpoints), linear combinations of vectors (e.g. centroids). The line at infinity, $\mathbf{l}_\infty$.
Similarity 4 dof	$\begin{bmatrix} sr_{11} & sr_{12} & t_x \\ sr_{21} & sr_{22} & t_y \\ 0 & 0 & 1 \end{bmatrix}$		Ratio of lengths, angle. The circular points, **I**, **J** (see section 2.7.3).
Euclidean 3 dof	$\begin{bmatrix} r_{11} & r_{12} & t_x \\ r_{21} & r_{22} & t_y \\ 0 & 0 & 1 \end{bmatrix}$		Length, area

p.44, R. Hartley and A. Zisserman. *Multiple View Geometry in Computer Vision*

Projective Transformations & Other Transformations of 3D Space

Group	Matrix	Distortion	Invariant properties
Projective 15 dof	$\begin{bmatrix} A & t \\ v^T & v \end{bmatrix}$		Intersection and tangency of surfaces in contact. Sign of Gaussian curvature.
Affine 12 dof	$\begin{bmatrix} A & t \\ 0^T & 1 \end{bmatrix}$		Parallelism of planes, volume ratios, centroids. The plane at infinity, π_∞, (see section 3.5).
Similarity 7 dof	$\begin{bmatrix} sR & t \\ 0^T & 1 \end{bmatrix}$		The absolute conic, Ω_∞, (see section 3.6).
Euclidean 6 dof	$\begin{bmatrix} R & t \\ 0^T & 1 \end{bmatrix}$		Volume.

p.78, R. Hartley and A. Zisserman. *Multiple View Geometry in Computer Vision*

Generalizing

Special Orthogonal & Special Euclidean Lie Algebras

- SO(n): R**o**tations

$$SO(n) = \{R \in \mathbb{R}^{n\times n} : RR^T = I, \det R = +1\}.$$

$$\exp(\widehat{\omega}\theta) = e^{\widehat{\omega}\theta} = I + \theta\widehat{\omega} + \frac{\theta^2}{2!}\widehat{\omega}^2 + \frac{\theta^3}{3!}\widehat{\omega}^3 + \ldots$$

- SE(n): Transformations of EUCLIDEAN space

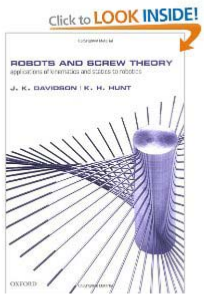
$$SE(n) := \mathbb{R}^n \times SO(n).$$

$$SE(3) = \{(p, R) : p \in \mathbb{R}^3, R \in SO(3)\} = \mathbb{R}^3 \times SO(3).$$

Screw Displacements

- Comes from the notion that all motion can be viewed as a rotation (Rodrigues formula)

- Define a vector along the axis of motion (screw vector)
  - Rotation (screw angle)
  - Translation (pitch)
  - Summations → via the screw triangle!



Unit Quaternion (ϵ_0, ϵ_1, ϵ_2, ϵ_3) [1]

- Does not suffer from singularities

$$\epsilon \equiv \epsilon_0 + \left(\epsilon_1 \hat{\mathbf{i}} + \epsilon_2 \hat{\mathbf{j}} + \epsilon_3 \hat{\mathbf{k}}\right)$$

- Uses a "4-number" to represent orientation

$$ii = jj = kk = -1$$
$$ij = k, jk = i, ki = j, ji = -k, kj = -1, ik = -j$$

- Product:

$$\begin{aligned}\mathbf{ab} \;=\; & (a_0 b_0 - a_1 b_1 - a_2 b_2 + a_3 b_3) \\ & + (a_0 b_1 + a_1 b_0 + a_2 b_3 - a_3 b_2)\,\hat{i} \\ & + (a_0 b_2 + a_2 b_0 + a_3 b_1 + a_1 b_3)\,\hat{j} \\ & + (a_0 b_3 + a_3 b_0 + a_1 b_2 - a_2 b_1)\,\hat{k}\end{aligned}$$

- Conjugate:

$$\tilde{\epsilon} \equiv \epsilon_0 - \epsilon_1 \hat{\mathbf{i}} - \epsilon_2 \hat{\mathbf{j}} - \epsilon_3 \hat{\mathbf{k}}$$
$$\epsilon\tilde{\epsilon} = \tilde{\epsilon}\epsilon = \epsilon_0^2 + \epsilon_1^2 + \epsilon_2^2 + \epsilon_3^2$$

Unit Quaternion [2]: Describing Orientation

- Set $\epsilon_0 = 0$
  Then $\mathbf{p}=(p_x,p_y,p_z)$ → $\quad \mathrm{p} = p_x\hat{\mathrm{i}} + p_y\hat{\mathrm{j}} + p_z\hat{\mathrm{k}}$

- Then given ϵ
  the operation $\epsilon \mathbf{p} \tilde{\epsilon}$: rotates $\mathbf{p}$ about $(\epsilon_1, \epsilon_2, \epsilon_3)$

- Unit Quaternion → Rotation Matrix

$$\mathbf{R} = \begin{pmatrix} 1-2\left(\epsilon_2^2+\epsilon_3^2\right) & 2\left(\epsilon_1\epsilon_2-\epsilon_0\epsilon_3\right) & 2\left(\epsilon_1\epsilon_3-\epsilon_0\epsilon_2\right) \\ 2\left(\epsilon_1\epsilon_2-\epsilon_0\epsilon_3\right) & 1-2\left(\epsilon_1^2+\epsilon_3^2\right) & 2\left(\epsilon_2\epsilon_3-\epsilon_0\epsilon_1\right) \\ 2\left(\epsilon_1\epsilon_3-\epsilon_0\epsilon_2\right) & 2\left(\epsilon_2\epsilon_3-\epsilon_0\epsilon_1\right) & 1-2\left(\epsilon_1^2+\epsilon_2^2\right) \end{pmatrix}$$

Direction Cosine

- Uses the Direction Cosines (read dot products) of the Coordinate Axes of the moving frame with respect to the fixed frame

$$^{A}\mathbf{u} = u_x\mathbf{i} + u_y\mathbf{j} + u_z\mathbf{k}$$
$$^{A}\mathbf{v} = v_x\mathbf{i} + v_y\mathbf{j} + v_z\mathbf{k}$$
$$^{A}\mathbf{w} = w_x\mathbf{i} + w_y\mathbf{j} + w_z\mathbf{k}$$

- It forms a rotation matrix!

$$\begin{array}{c|c} {}^{A}_{B}R & \quad (b_x)\,\hat{\imath}_B \quad\; (b_y)\,\hat{\jmath}_B \quad\; (b_z)\,\hat{k}_B \\ \hline \begin{array}{c} (a_x)\,\hat{\imath}_A \\ (a_y)\,\hat{\jmath}_A \\ (a_z)\,\hat{k}_A \end{array} & \begin{bmatrix} \hat{\imath}_B\cdot\hat{\imath}_A & \hat{\jmath}_B\cdot\hat{\imath}_A & \hat{k}_B\cdot\hat{\imath}_A \\ \hat{\imath}_B\cdot\hat{\jmath}_A & \hat{\jmath}_B\cdot\hat{\jmath}_A & \hat{k}_B\cdot\hat{\jmath}_A \\ \hat{\imath}_B\cdot\hat{k}_A & \hat{\jmath}_B\cdot\hat{k}_A & \hat{k}_B\cdot\hat{k}_A \end{bmatrix} \end{array}$$

KINEMATICS …

Forward Kinematics [1]

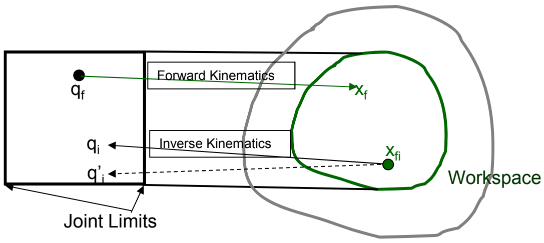
- Forward kinematics is the process of chaining homogeneous transforms together. For example to:
  - Find the articulations of a mechanism, or
  - the fixed transformation between two frames which is known in terms of linear and rotary parameters.
- Calculates the final position from the **machine** (**<u>joint variables</u>**)
- Unique for an open kinematic chain (**serial arm**)
- "Complicated" (multiple solutions, etc.) for a closed kinematic chain (**parallel arm**)

Forward Kinematics [2]

- Can think of this as "spaces":
  - Workspace (x,y,z,α,β,γ):
    The robot's position & orientation
  - Joint space ($\theta_1 \dots \theta_n$):
    A state-space vector of joint variables

$$\vec{\mathbf{x}} = \begin{bmatrix} \vec{\mathbf{p}} \\ \vec{\Theta} \end{bmatrix}$$

$$\vec{\mathbf{q}} = \begin{bmatrix} q_1 \\ \vdots \\ q_n \end{bmatrix}$$

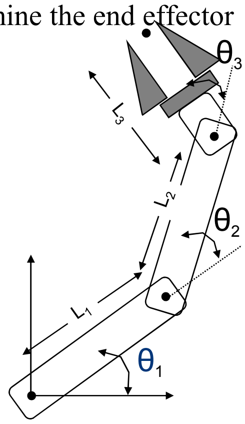


Forward Kinematics [3]

- Consider a planar RRR manipulator
- Given the joint angles and link lengths, we can determine the end effector pose:

$$x = L_1 \cos\theta_1 + L_2 \cos(\theta_1 + \theta_2) + \dots$$
$$L_3 \cos(\theta_1 + \theta_2 + \theta_3)$$

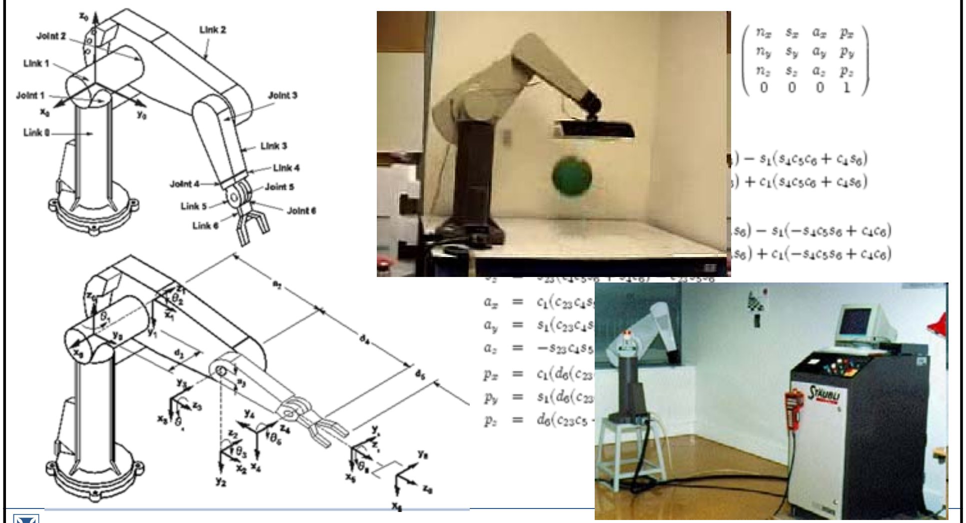
$$y = L_1 \sin\theta_1 + L_2 \sin(\theta_1 + \theta_2) + \dots$$
$$L_3 \sin(\theta_1 + \theta_2 + \theta_3)$$



- This isn't too difficult to determine for a simple, planar manipulator. BUT …

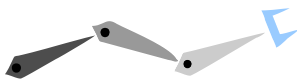
Forward Kinematics [4]: The PUMA 560!

- What about a more complicated mechanism?



Inverse Kinematics

- Forward: angles → position
  $\mathbf{x} = f(\boldsymbol{\theta})$
- Inverse: position → angles
  $\boldsymbol{\theta} = f^{-1}(\mathbf{x})$
- Analytic Approach



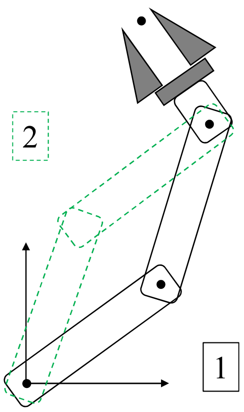
- Numerical Approaches:
  - Jacobian: $J = \frac{\delta x}{\delta q} \rightarrow \delta q \approx J^{-1}\delta x$
  - J^T Approximation: $\tau = J^T \cdot \mathbf{F} \rightarrow \Delta q \approx J^T \Delta x$
    - Slotine & Sheridan method
  - Cyclical Coordinate Descent

Inverse Kinematics

- Inverse Kinematics is the problem of finding the joint parameters given only the values of the homogeneous transforms which model the mechanism (i.e., the pose of the end effector)
- Solves the problem of where to drive the joints in order to get the hand of an arm or the foot of a leg in the right place
- In general, this involves the solution of a set of simultaneous, non-linear equations
- Hard for serial mechanisms, easy for parallel

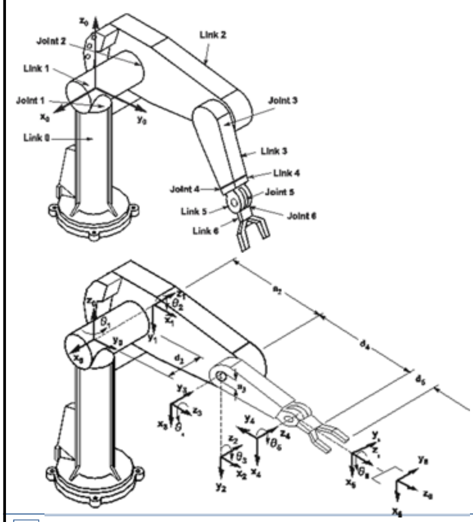
Multiple Solutions

- There will often be multiple solutions for a particular inverse kinematic analysis
- Consider the three link manipulator shown. Given a particular end effector pose, two solutions are possible
- The choice of solution is a function of proximity to the current pose, limits on the joint angles and possible obstructions in the workspace



Inverse kinematics

- What about a more complicated mechanism?



$$^0T_6 = {}^0T_1\,{}^1T_2\,{}^2T_3\,{}^3T_4\,{}^4T_5\,{}^5T_6 = \begin{pmatrix} n_x & s_x & a_x & p_x \\ n_y & s_y & a_y & p_y \\ n_z & s_z & a_z & p_z \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

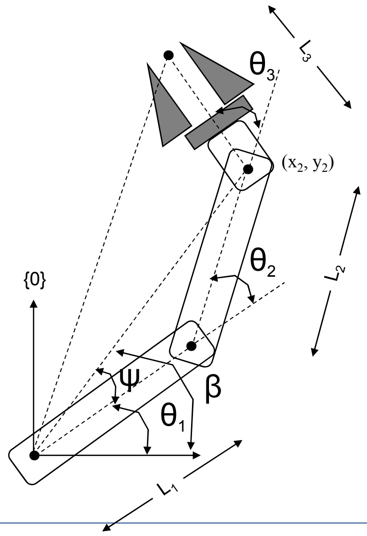
$$\begin{aligned}
n_x &= c_1(c_{23}(c_4c_5c_6 - s_4s_6) - s_{23}s_5c_6) - s_1(s_4c_5c_6 + c_4s_6) \\
n_y &= s_1(c_{23}(c_4c_5c_6 - s_4s_6) - s_{23}s_5c_6) + c_1(s_4c_5c_6 + c_4s_6) \\
n_z &= -s_{23}(c_4c_5c_6 - s_4s_6) - c_{23}s_5c_6 \\
s_x &= c_1(-c_{23}(c_4c_5s_6 + s_4c_6) + s_{23}s_5s_6) - s_1(-s_4c_5s_6 + c_4c_6) \\
s_y &= s_1(-c_{23}(c_4c_5s_6 + s_4c_6) + s_{23}s_5s_6) + c_1(-s_4c_5s_6 + c_4c_6) \\
s_z &= s_{23}(c_4c_5s_6 + s_4c_6) - c_{23}s_5s_6 \\
a_x &= c_1(c_{23}c_4s_5 + s_{23}c_5) - s_1s_4s_5 \\
a_y &= s_1(c_{23}c_4s_5 + s_{23}c_5) + c_1s_4s_5 \\
a_z &= -s_{23}c_4s_5 + c_{23}c_5) \\
p_x &= c_1(d_6(c_{23}c_4s_5 + s_{23}c_5) + s_{23}d_4 + a_3c_{23} + a_2c_2) - s_1(d_6s_4s_5 + d_2) \\
p_y &= s_1(d_6(c_{23}c_4s_5 + s_{23}c_5) + s_{23}d_4 + a_3c_{23} + a_2c_2) + c_1(d_6s_4s_5 + d_2) \\
p_z &= d_6(c_{23}c_5 - s_{23}c_4s_5) + c_{23}d_4 - a_3s_{23} - a_2s_2
\end{aligned}$$

Solution Methods

- Unlike with systems of linear equations, there are no general algorithms that may be employed to solve a set of nonlinear equation
- **Closed-form** and **numerical** methods exist
- We will concentrate on analytical, closed-form methods
- These can be characterized by two methods of obtaining a solution: **algebraic** and **geometric**

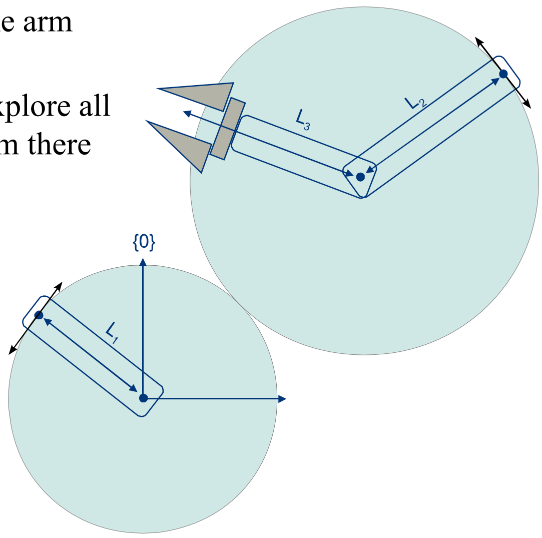
Inverse Kinematics: Geometrical Approach

- We can also consider the geometric relationships defined by the arm



Inverse Kinematics: Geometrical Approach [2]

- We can also consider the geometric relationships defined by the arm
- Start with what is fixed, explore all geometric possibilities from there



Inverse Kinematics: Algebraic Approach

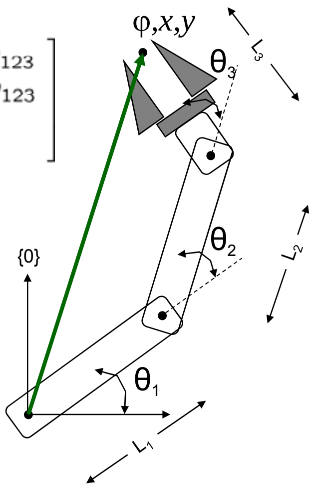
- We have a series of equations which define this system
- Recall, from Forward Kinematics:

$$
{}^{0}T_3 = \begin{bmatrix} c_{\theta_{123}} & -s_{\theta_{123}} & 0 & L_1 c_{\theta_1} + L_2 c_{\theta_{12}} + L_3 c_{\theta_{123}} \\ s_{\theta_{123}} & c_{\theta_{123}} & 0 & L_1 s_{\theta_1} + L_2 s_{\theta_{12}} + L_3 s_{\theta_{123}} \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

- The end-effector pose is given by

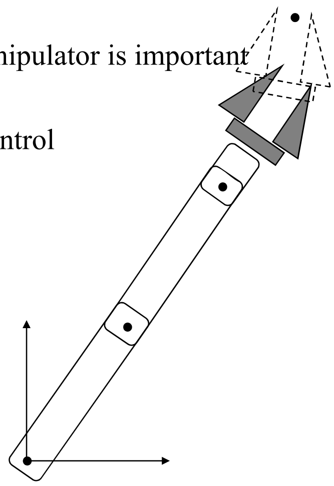
$$
{}^{0}T_3 = \begin{bmatrix} c_\phi & -s_\phi & 0 & x \\ s_\phi & c_\phi & 0 & y \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

- Equating terms gives us a set of algebraic relationships



No Solution - Singularity

- Singular positions:
- An understanding of the workspace of the manipulator is important
- There will be poses that are not achievable
- There will be poses where there is a loss of control
- Singularities also occur when the manipulator loses a DOF
  - This typically happens when joints are aligned
  - **det[Jacobian]=0**



Denavit Hartenberg [DH] Notation

- J. Denavit and R. S. Hartenberg first proposed the use of homogeneous transforms for articulated mechanisms

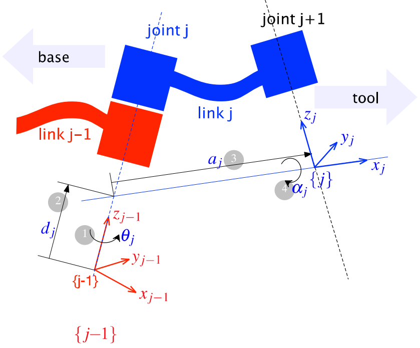
  (But B. Roth, introduced it to robotics)

- A kinematics “short-cut” that reduced the number of parameters by adding a structure to frame selection

- For two frames positioned in space, the first can be moved into coincidence with the second by a sequence of 4 operations:
  - rotate around the x_{i-1} axis by an angle α_i
  - translate along the x_{i-1} axis by a distance a_i
  - translate along the new z axis by a distance d_i
  - rotate around the new z axis by an angle θ_i

Denavit-Hartenberg Convention

- link length a_i the offset distance between the z_{i-1} and z_i axes along the x_i axis;
- link twist α_i the angle from the z_{i-1} axis to the z_i axis about the x_i axis;



Art c/o P. Corke

- link offset d_i the distance from the origin of frame i-1 to the x_i axis along the z_{i-1} axis;
- joint angle θ_i the angle between the x_{i-1} and x_i axes about the z_{i-1} axis.

DH: Where to place frame?

1. Align an axis along principal motion
   1. <u>Rotary (**R**)</u>: align rotation axis along the <u>z axis</u>
   2. <u>Prismatic (**P**)</u>: align slider travel along <u>x axis</u>

2. Orient so as to position x axis towards next frame

3. θ (rot z) → d (trans z) → a (trans x) → α (rot x)

Denavit-Hartenberg → Rotation Matrix

- Each transformation is a product of 4 "basic" transformations (instead of 6)

$$
{}^{i-1}A_i = Rot_{z,\theta_i} Trans_{z,d_i} Trans_{x,a_i} Rot_{x,\alpha_i}
$$

$$
= \begin{bmatrix} c_{\theta_i} & -s_{\theta_i} & 0 & 0 \\ s_{\theta_i} & c_{\theta_i} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & a_i \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdots
$$

$$
\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & c_{\alpha_i} & -s_{\alpha_i} & 0 \\ 0 & s_{\alpha_i} & c_{\alpha_i} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

$$
= \begin{bmatrix} c_{\theta_i} & -s_{\theta_i} c_{\alpha_i} & s_{\theta_i} s_{\alpha_i} & a_i c_{\theta_i} \\ s_{\theta_i} & c_{\theta_i} c_{\alpha_i} & -c_{\theta_i} s_{\alpha_i} & a_i s_{\theta_i} \\ 0 & s_{\alpha_i} & c_{\alpha_i} & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

DH Example [1]: RRR Link Manipulator

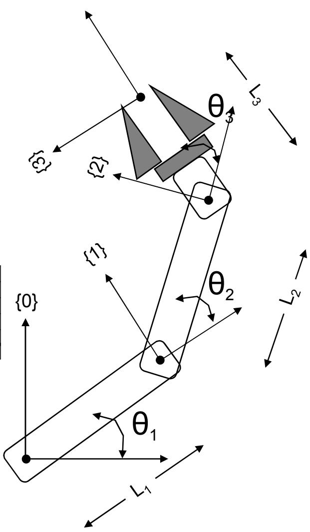
1. Assign the frames at the joints …
2. Fill DH Table …

Link	a_i	α_i	d_i	θ_i
1	L_1	0	0	θ_1
2	L_2	0	0	θ_2
3	L_3	0	0	θ_3

$$
{}^0A_1 = \begin{bmatrix} c_{\theta_1} & -s_{\theta_1} & 0 & L_1 c_{\theta_1} \\ s_{\theta_1} & c_{\theta_1} & 0 & L_1 s_{\theta_1} \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}
{}^1A_2 = \begin{bmatrix} c_{\theta_2} & -s_{\theta_2} & 0 & L_2 c_{\theta_2} \\ s_{\theta_2} & c_{\theta_2} & 0 & L_2 s_{\theta_2} \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}
{}^2A_3 = \begin{bmatrix} c_{\theta_3} & -s_{\theta_3} & 0 & L_3 c_{\theta_3} \\ s_{\theta_3} & c_{\theta_3} & 0 & L_3 s_{\theta_3} \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

$$
{}^0T_3 = {}^0A_1 \, {}^1A_2 \, {}^2A_3
$$

$$
= \begin{bmatrix} c_{\theta_{123}} & -s_{\theta_{123}} & 0 & L_1 c_{\theta_1} + L_2 c_{\theta_{12}} + L_3 c_{\theta_{123}} \\ s_{\theta_{123}} & c_{\theta_{123}} & 0 & L_1 s_{\theta_1} + L_2 s_{\theta_{12}} + L_3 s_{\theta_{123}} \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$



DH Example [2]: RRP Link Manipulator

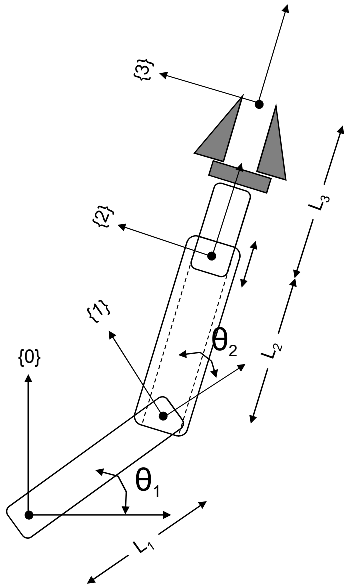
1. Assign the frames at the joints …
2. Fill DH Table …

Link	a_i	α_i	d_i	θ_i
1	L_1	0	0	θ_1
2	L_2	0	0	θ_2
3	L_3	0	0	0

$$^0A_1 = \begin{bmatrix} c_{\theta_1} & -s_{\theta_1} & 0 & L_1c_{\theta_1} \\ s_{\theta_1} & c_{\theta_1} & 0 & L_1s_{\theta_1} \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \; ^1A_2 = \begin{bmatrix} c_{\theta_2} & -s_{\theta_2} & 0 & L_2c_{\theta_2} \\ s_{\theta_2} & c_{\theta_2} & 0 & L_2s_{\theta_2} \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \; ^2A_3 = \begin{bmatrix} 1 & 0 & 0 & L_3 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

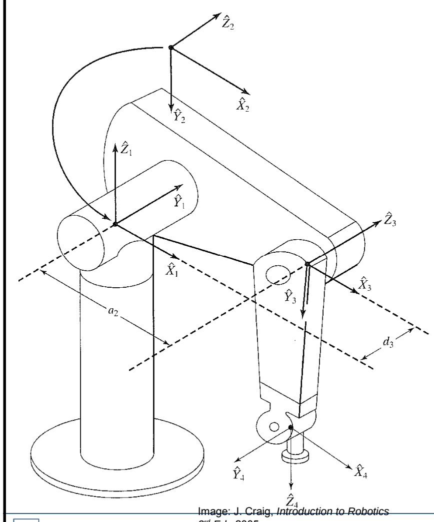
$$^0T_3 = {}^0A_1\,{}^1A_2\,{}^2A_3$$

$$= \begin{bmatrix} c_{\theta_{12}} & -s_{\theta_{12}} & 0 & L_1c_{\theta_1} + (L_2+L_3)c_{\theta_{12}} \\ s_{\theta_{12}} & c_{\theta_{12}} & 0 & L_1s_{\theta_1} + (L_2+L_3)s_{\theta_{12}} \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$



DH Example [3]: Puma 560

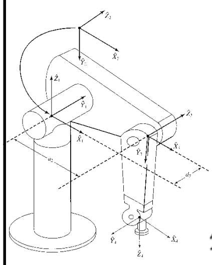
- "Simple" 6R robot exercise for the reader …



Link	a_i	α_i	d_i	θ_i
1	0	0	0	θ_1
2	0	$-\pi/2$	0	θ_2
3	L_2	0	D_3	θ_3
4	L_3	$-\pi/2$	D_4	θ_4
5	0	$\pi/2$	0	θ_5
6	0	$-\pi/2$	0	θ_6

Image: J. Craig, *Introduction to Robotics* 3rd Ed., 2005

DH Example [3]: Puma 560 [2]



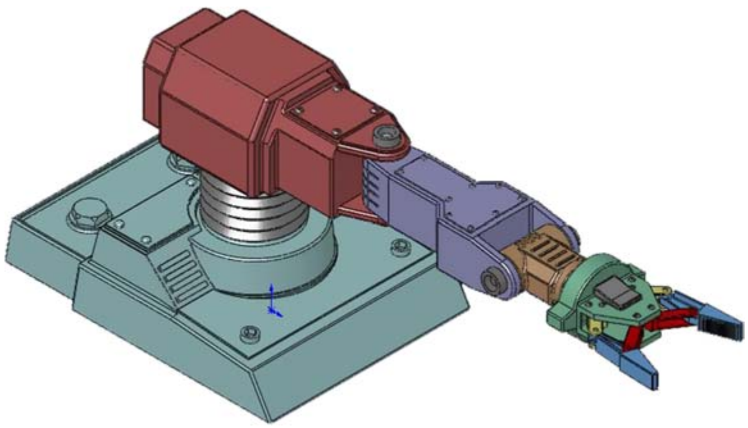
$$
{}^{0}A_1 = \begin{bmatrix} c_1 & -s_1 & 0 & 0 \\ s_1 & c_1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}
\quad
{}^{1}A_2 = \begin{bmatrix} c_2 & -s_2 & 0 & 0 \\ 0 & 0 & 1 & d_2 \\ -s_2 & -c_2 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

$$
{}^{2}A_3 = \begin{bmatrix} c_3 & -s_3 & 0 & L_2 \\ s_3 & c_3 & 0 & 0 \\ 0 & 0 & 1 & d_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}
\quad
{}^{3}A_4 = \begin{bmatrix} c_4 & -s_4 & 0 & L_3 \\ 0 & 0 & 1 & d_4 \\ -s_4 & -c_4 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

$$
{}^{4}A_5 = \begin{bmatrix} c_4 & -s_5 & 0 & L_3 \\ 0 & 0 & 1 & d_4 \\ -s_5 & -c_5 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}
\quad
{}^{5}A_6 = \begin{bmatrix} c_6 & -s_6 & 0 & L_3 \\ 0 & 0 & -1 & 0 \\ -s_6 & -c_6 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

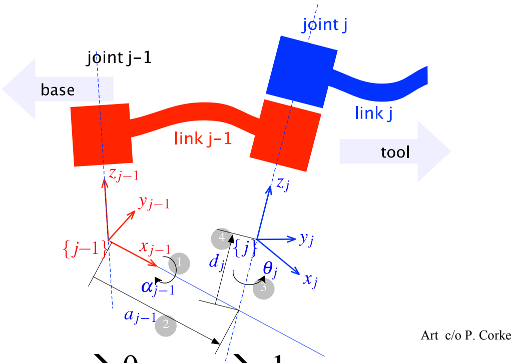
$$
{}^{0}T_6 = {}^{0}A_1 {}^{1}A_2 {}^{2}A_3 {}^{3}A_4 {}^{4}A_5 {}^{5}A_6
$$

Demonstration: Matlab & Solidworks



Modified DH

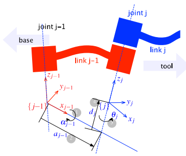
- Made "popular" by Craig's *Intro. to Robotics* book
- Link coordinates attached to the near by joint



- a (trans *x-1*) → α (rot *x-1*) → θ (rot *z*) → d (trans *z*)

Modified DH [2]

- Gives a similar result
  (but it's not commutative)



$$\Rightarrow {}^{i-1}A_i = R_x(\alpha_{i-1})\, T_x(a_{i-1})\, R_z(\theta_i)\, T_x(d_i)$$

- Refactoring Standard → to Modified

$$\underbrace{\{R_z(\theta_1)\,T_z(d_1)\,T_x(a_1)\,R_x(\alpha_1)\}}_{\text{DH}_1}\cdot\underbrace{\{R_z(\theta_2)\,T_z(d_2)\,T_x(a_2)\,R_x(\alpha_2)\}}_{\text{DH}_2}\cdot\underbrace{\{R_z(\theta_3)\,T_z(d_3)\}}_{\text{End Effector}}$$

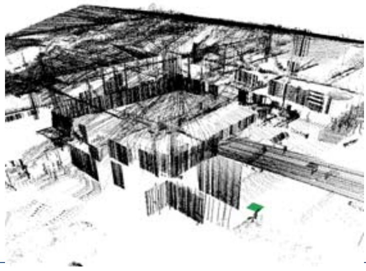
$$=\underbrace{\{R_z(\theta_1)\,T_z(d_1)\}}_{\text{Base}}\cdot\underbrace{\{T_x(a_1)\,R_x(\alpha_1)\,R_z(\theta_2)\,T_z(d_2)\}}_{\text{MDH}_1}\cdot\underbrace{\{T_x(a_2)\,R_x(\alpha_2)\,R_z(\theta_3)\,T_z(d_3)\}}_{\text{MDH}_2}$$

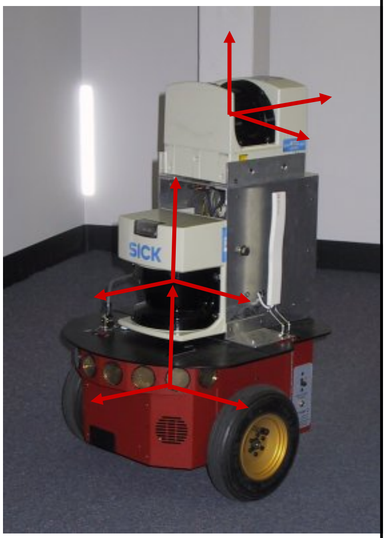
Mobile Platforms

- The preceding kinematic relationships are also important in mobile applications
- When we have sensors mounted on a platform, we need the ability to translate from the sensor frame into some world frame in which the vehicle is operating
- Should we just treat this as a P(*) mechanism?

Mobile Platforms [2]

- We typically assign a frame to the base of the vehicle
- Additional frames are assigned to the sensors
- We will develop these techniques in coming lectures





Summary

- Many ways to view a rotation
  - Rotation matrix
  - Euler angles
  - Quaternions
  - Direction Cosines
  - Screw Vectors
- Homogenous transformations
  - Based on homogeneous coordinates

Cool Robotics Share