Motion Planning & Control

METR 4202: Advanced Control & **Robotics**

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Lecture # 11

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RoboticsCourseWare Contributor

Schedule

Week	Date	Lecture (F: 9-10:30, 42-212)
1	26-Jul	Introduction
2	2-Aug	Representing Position & Orientation & State (Frames, Transformation Matrices & Affine Transformations)
3	9-Aug	Robot Kinematics
4	16-Aug	Robot Dynamics & Control
5	23-Aug	Robot Trajectories & Motion
6	30-Aug	Sensors & Measurement
7	6-Sep	Perception / Computer Vision
8	13-Sep	Localization and Navigation
9	20-Sep	State-Space Modelling
10	27-Sep	State-Space Control
	4-Oct	*Study break*
11	11-Oct	Motion Planning
12	**18-Oct**	**Motion Planning & Control**
13	25-Oct	Applications in Vision-based control (+ Prof. P. Corke) and in Industry (+ Prof. S. LaValle) & Course Review

Announcements: We're Working On It!

- **Take Home "Quiz":**
  - Via Platypus:
    - Scanning of Handwritten solutions okay
  - Answer **4 Questions (Out of 10)**
    - **Two of the questions will be selected for you (via email)**
    - **Additional two:** you select from the 8 remaining questions
  - You have 48-hours from when you first login
  - Due by **Nov 15** (end of finals period)
  - Will go live this weekend (Still debugging random selection code)

- Cool Robotics Share Site
  ➔ http://metr4202.tumblr.com/

Cool
Robotics
Video
Share

Announcements: Guest Lecturers Next Week!

- **Peter Corke:**
  - Vision-Based Servo Control

- **Latzi Marian:**
  - Applications of METR4202 (Vision Processing) to Food Processing Automation & PLC

- **Steve LaValle:**
  - Co-Inventor of the RRT
  - Now CTO for Oculus Rift

Cool
Robotics
Video
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Motion
PLANNING
& Uncertainty

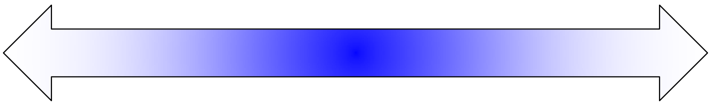
(Slides C/O Hanna Kurniawati)

Deterministic motion planning vs motion planning under uncertainty

- Deterministic motion planning
  - Find a valid path between two configurations in order to accomplish a task, given:
  - No control error.
  - No sensing.
  - Know the operating environment perfectly.
- Motion planning under uncertainty (today)
  - Find a motion strategy to accomplish a task, where there's a combination of:
  - Control error.
  - Sensing error.
  - Partially / unknown operating environment.

Today: wonderful world of ~~“perfect”~~ robotics

- In particular: Motion planning under uncertainty.
  - ✓ What is it.
  - ☐ How difficult to solve it.
  - ☐ Several approaches:



Conservative Probabilistic Reactive

Problem hardness

- Finding a motion strategy for:
  - A point robot operating in 3D environment, where obstacles are planar walls.
  - To move from a known initial configuration to a point in a given goal region.
  - Control error: Bounded velocity error.
  - Sensing error: Bounded localization error.

is PSPACE-hard [Natarajan'86].

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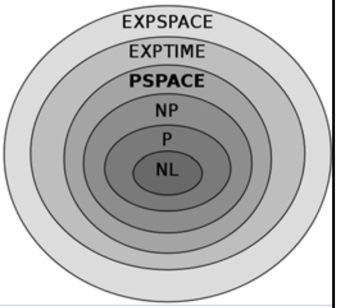
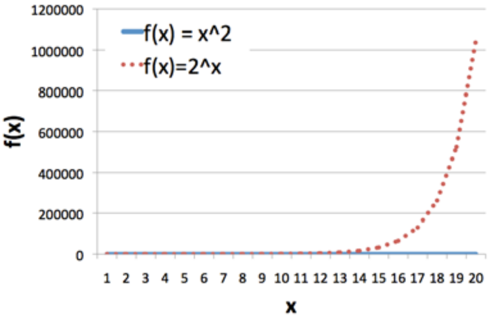


A little bit on computational complexity 1/3

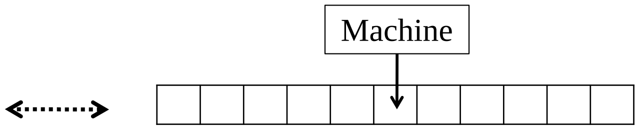
- Algorithms are **not** made to be used only once & are **not** made to be used for only one particular problem.
- How long does it take for the algorithm to find the solution when the input size increases ?
  - In particular, is it polynomial or exponential ?



A little bit on computational complexity 2/3

Today’s computer

(Deterministic) Turing machine

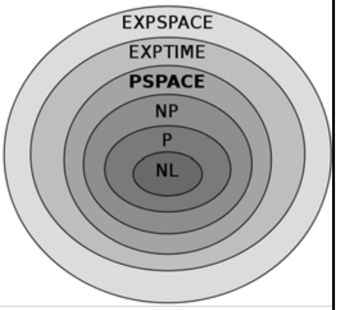


Non-Deterministic Turing machine
It can generate multiple possible program executions at once.
Same capability as Turing machine, but can get things done faster.

A little bit on computational complexity 3/3

- P: Can be solved in polynomial time in Turing machine.
- NP: Can be solved in polynomial time on a non-deterministic Turing machine.

  → Verifiable in polynomial time in today's computer.
- PSPACE: Can be solved using polynomial space in Turing machine.



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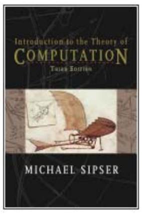
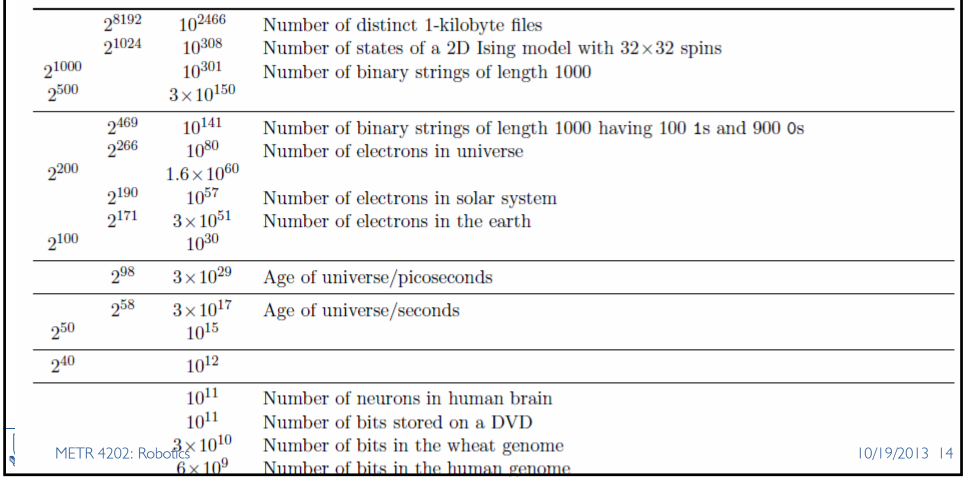
Want to know more ?
Introduction to the Theory of Computation
by Michael Sipser.



But Intel Giveth!

- “Moore’s Law” is exponential (at best!)
- These problems ∝ factorial!
- Some Numbers: (From: D. MacKay, Information Theory, Inference, and Learning Algorithms)



	2^{8192}	10^{2466}	Number of distinct 1-kilobyte files
	2^{1024}	10^{308}	Number of states of a 2D Ising model with 32×32 spins
2^{1000}		10^{301}	Number of binary strings of length 1000
2^{500}		3×10^{150}	
	2^{469}	10^{141}	Number of binary strings of length 1000 having 100 1s and 900 0s
	2^{266}	10^{80}	Number of electrons in universe
2^{200}		1.6×10^{60}	
	2^{190}	10^{57}	Number of electrons in solar system
	2^{171}	3×10^{51}	Number of electrons in the earth
2^{100}		10^{30}	
	2^{98}	3×10^{29}	Age of universe/picoseconds
	2^{58}	3×10^{17}	Age of universe/seconds
2^{50}		10^{15}	
2^{40}		10^{12}	
		10^{11}	Number of neurons in human brain
		10^{11}	Number of bits stored on a DVD
		3×10^{10}	Number of bits in the wheat genome
		6×10^{9}	Number of bits in the human genome

Problem hardness

- Finding a motion strategy for:
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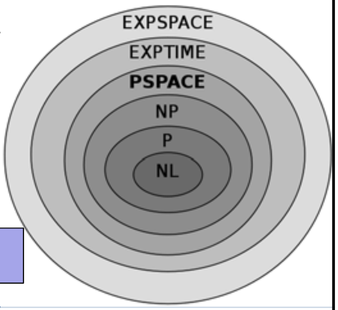
  input size: number of planar walls.



Ok, it’s hard… So, what should we do ?

Today: wonderful world of ~~“perfect”~~ robotics

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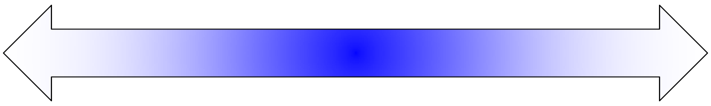
  - ✓ What is it.
  - ✓ How difficult to solve it.
  - ☐ Several approaches:

    Methods: Algorithms vs heuristics

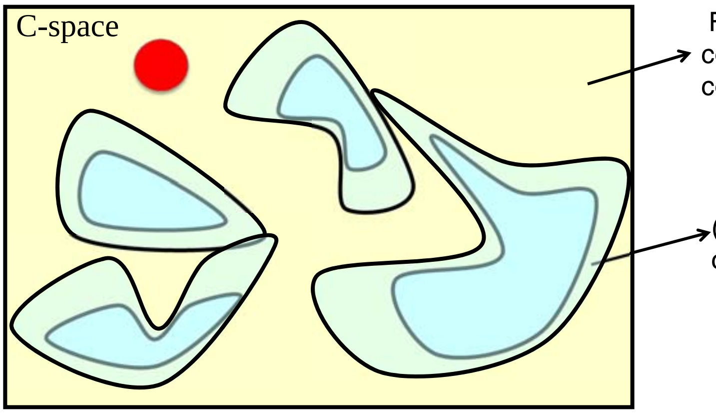


Conservative
Overestimate risk.

Probabilistic
Quantify uncertainty, to tradeoff risk w. achieving goal.

Reactive
Underestimate difficulty of achieving goal.

Simplest algorithm: Enlarging obstacles

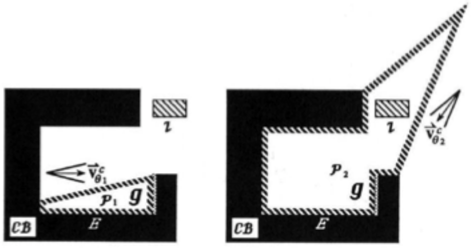


Free space (set of collision-free configurations).

Forbidden region (set of colliding configurations).

Pre-image backchaining algorithm

- Uncertainty in motion & sensing.
  - Ability to recognize if it's in a particular region.
- Motion command: (control input, termination condition).
- Pre-image: Region of C-space, where a motion command is guarantee to reach a given goal region, recognizably.



Ch. 10


ROBOT MOTION PLANNING

Jean-Claude Latombe

Imagine…

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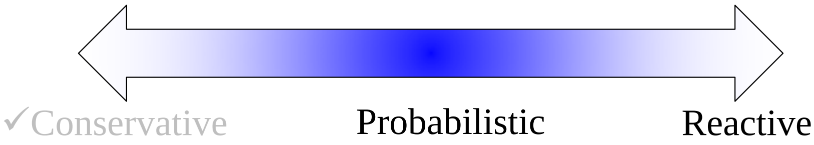
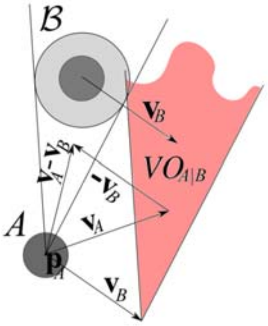


✓Conservative Probabilistic Reactive

Reactive greedy heuristic, based on Velocity obstacle

- Velocity obstacle



$$VO_{A|B} \quad v \,\Big|\, \quad t \quad 0\; p_A \quad t\; v \quad v_B \quad PosCol$$

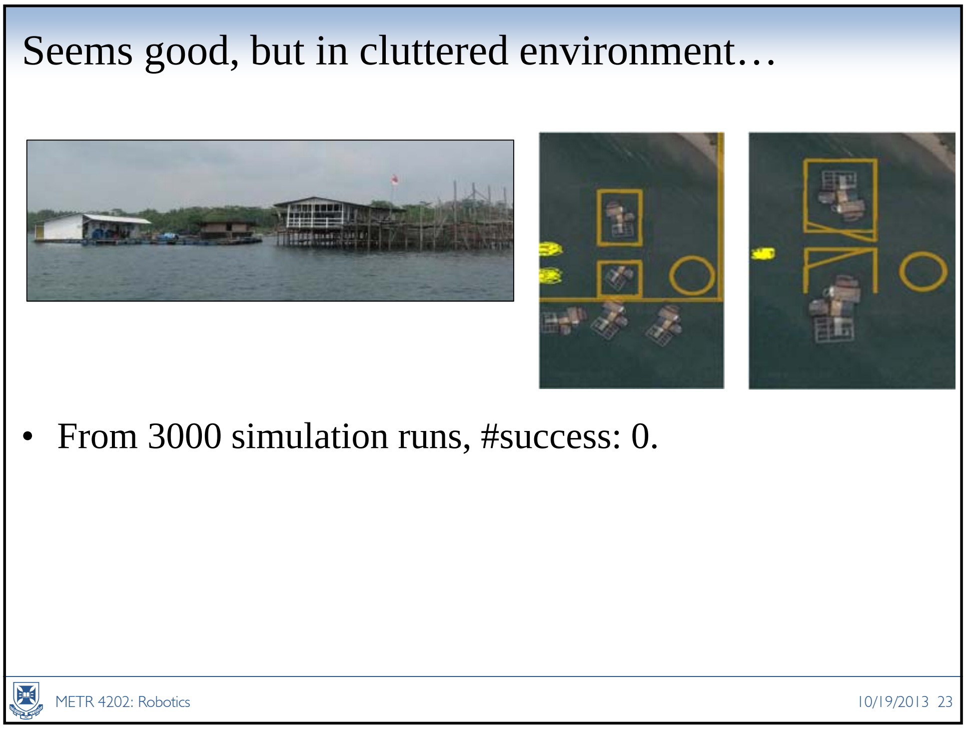
p_A : center of robot A.

$PosCol$: cone region, with tip p_A, covering B   A.

- Avoid velocity in VO, choose the one closest to goal.

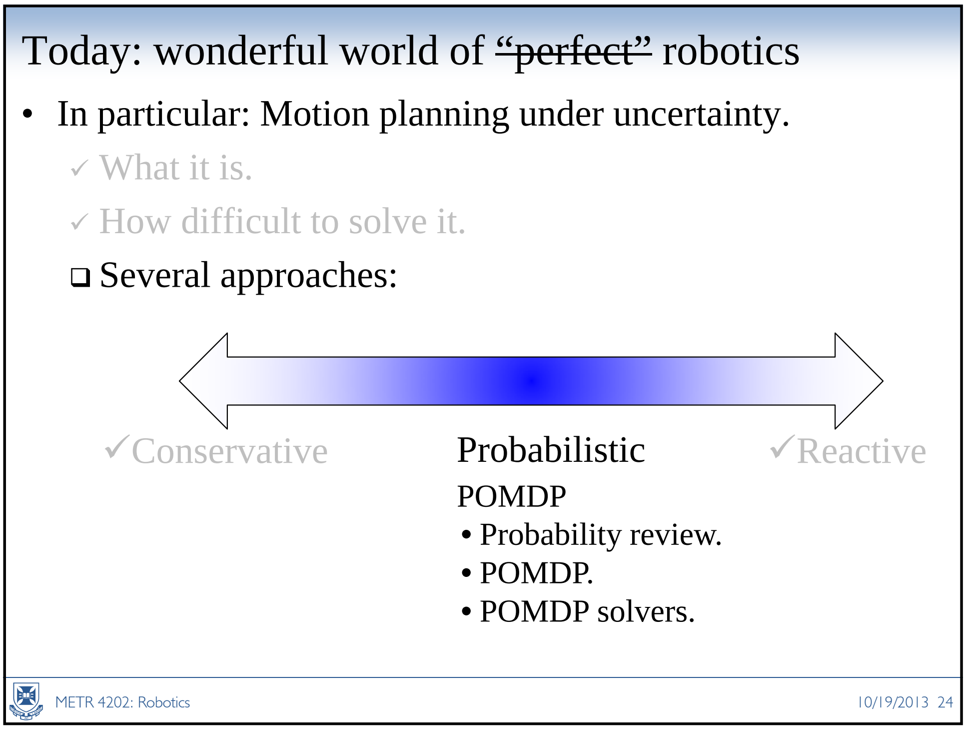
In open environment…

T. Bandyopadhyay & F. Hover, 2009.



Seems good, but in cluttered environment…

- From 3000 simulation runs, #success: 0.



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✓Conservative

Probabilistic

POMDP
- Probability review.
- POMDP.
- POMDP solvers.

✓Reactive

How to quantify uncertainty ? Probability to the rescue…

Bertsekas & Tsitsiklis, Introduction to Probability.

- FATHER(F): Nurse, what is the probability that the drug will work?
- NURSE (N): I hope it works, we'll know tomorrow.
- F: Yes, but what is the probability that it will?
- N: Each case is different, we have to wait.
- F: But let's see, out of a hundred patients that are treated under similar conditions, how many times would you expect it to work?
- N (somewhat annoyed): I told you, every person is different, for some it works, for some it doesn't.
- F (insisting): Then tell me, if you had to bet whether it will work or not, which side of the bet would you take?
- N (cheering up for a moment): I'd bet it will work.
- F (somewhat relieved): OK, now, would you be willing to lose two dollars if it doesn't work, and gain one dollar if it does?
- N (exasperated): What a sick thought! You are wasting my time!

Probability review 1/4: Probabilistic Modeling

- View:
  - Experiments with random outcome.
  - Quantifiable properties of the outcome.
- Three components:
  - Sample space: Set of all possible outcomes.
  - Events: Subsets of sample space.
  - Probability: Quantify how likely an event occurs.

Probability review 2/4: Probability

- Probability: A function that maps events to real numbers satisfying these axioms:
  1. Non-negativity: $P(E) \quad 0$, where E is an ever
  2. Normalization: $P(S) \quad 1$, where S is the sample space.
  3. Additivity of finite / countably infinite events.

$$P\left(\bigcup_{i \; 1}^{/n} E_i\right) \qquad \sum_{i \; 1}^{/n} P\left(E_i\right),$$

where E $_i$ are disjoint / mutually exclusive, i : natural number.

Probability review 3/4: Random Variables

- Interest is on numerical values associated w. samples, e.g.:
  - Sample 50 students enrolled in METR4202, what's the major of most of the students.
  - Roll a fair dice, get \$5 if the outcome is even, & loose \$5 if the outcome is odd.
- Random variable X is a function $X : S \quad Num.$
  - Num: countable set (e.g., integer) → discrete random variable.
  - Num: uncountable set (e.g., real) → continuous random variable.

Probability review 4/4: Characterizing Random Variables

- Cumulative distribution function (cdf)

$$F_X(x) = P(X \le x) = P(\{s \mid X(s) \le x, s \in S\})$$

- Discrete: Probability mass function (pmf)

$$f_X(x) = P(X = x)$$

- Continuous: Probability density function/probability distribution function (pdf)

$$f_X(x) = \frac{dF_X(x)}{dx} \; ; \; P(a \le X \le b) = \int_a^b f_X(x)\, dx$$

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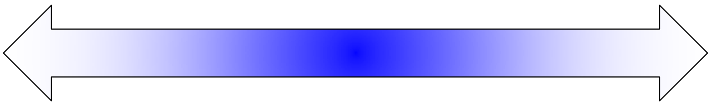

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