Motion Planning

METR 4202: Advanced Control & **Robotics**

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Lecture # 11

October 11, 2013

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Schedule

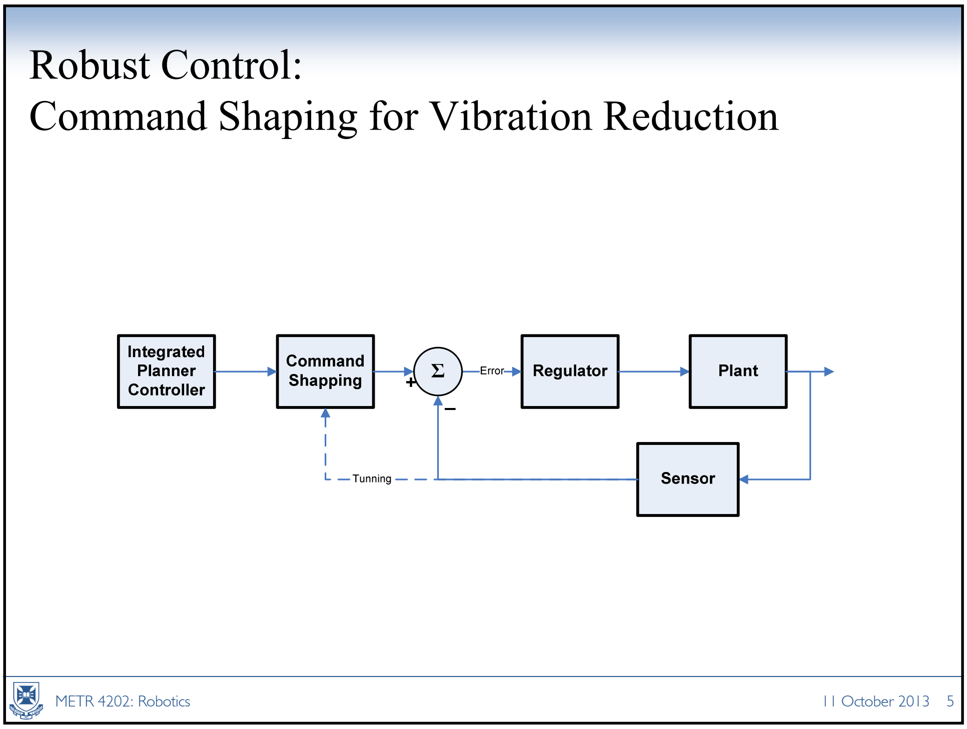
Week	Date	Lecture (F: 9-10:30, 42-212)
1	26-Jul	Introduction
2	2-Aug	Representing Position & Orientation & State (Frames, Transformation Matrices & Affine Transformations)
3	9-Aug	Robot Kinematics
4	16-Aug	Robot Dynamics & Control
5	23-Aug	Robot Trajectories & Motion
6	30-Aug	Sensors & Measurement
7	6-Sep	Perception / Computer Vision
8	13-Sep	Localization and Navigation
9	20-Sep	State-Space Modelling
10	27-Sep	State-Space Control
	4-Oct	*Study break*
11	**11-Oct**	**Motion Planning**
12	18-Oct	Vision-based control (+ Prof. P. Corke or Prof. M. Srinivasan)
13	25-Oct	Applications in Industry (+ Prof. S. LaValle) & Course Review

Announcements: We're Working On It!

- **Lab 3:**
  - Working on it!
  - Due: October 24 (!!), "Free Extension" to October 31.
- **Take Home "Quiz":**
  - Working on it!
  - Designing it so that you can do it in less than 24 hours if you know the question
  - Via Platypus: Scanning of Handwritten solutions okay
- Cool Robotics Share Site
  ➔ http://metr4202.tumblr.com/

Cool Robotics Video Share

From Last Week: Command Shaping



Robust Control:
Command Shaping for Vibration Reduction

Integrated Planner Controller

Command Shapping

Σ

+

−

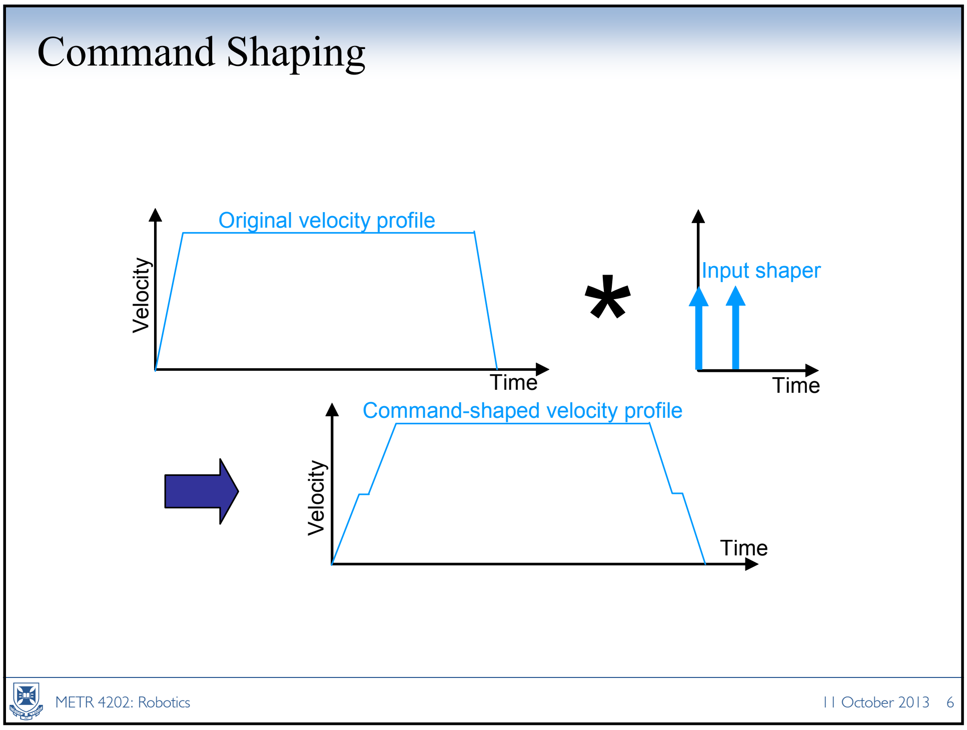
Error

Regulator

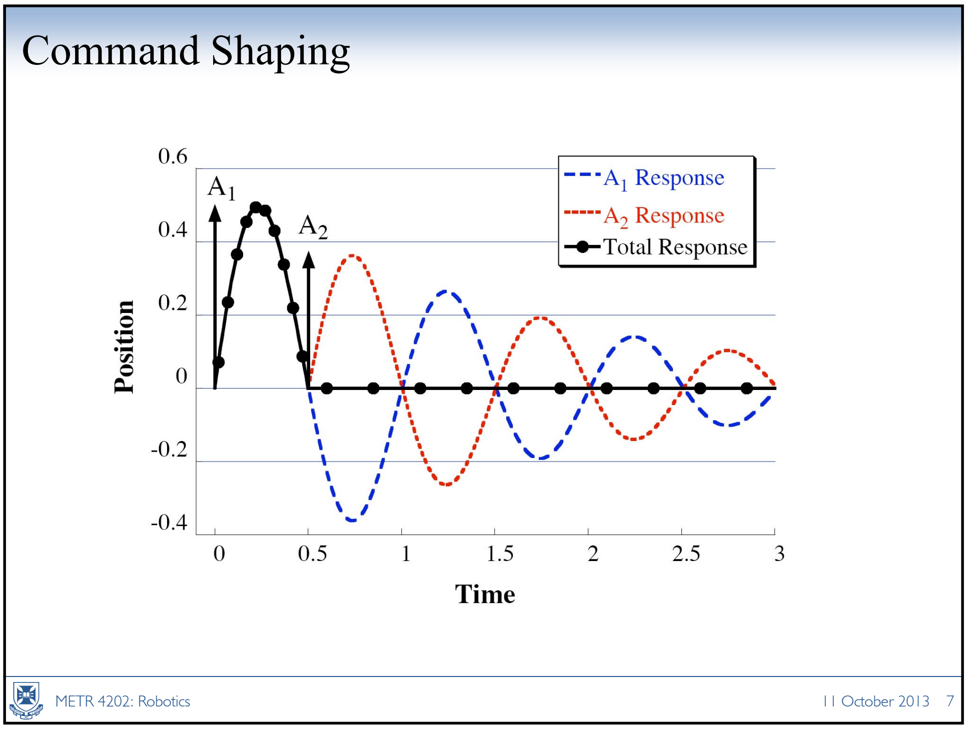
Plant

Sensor

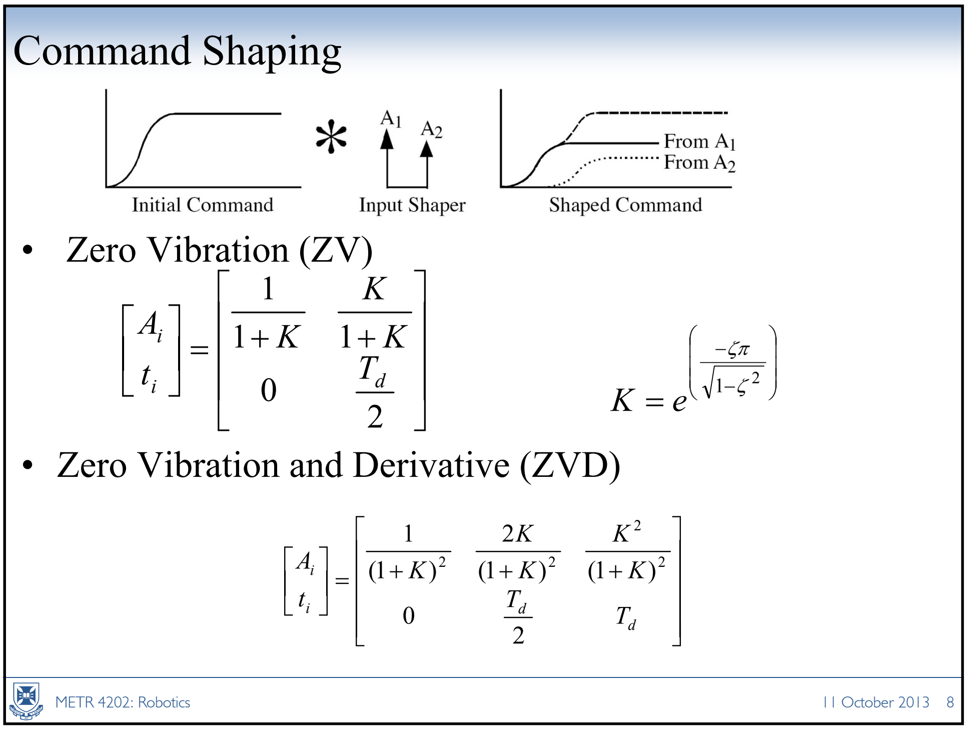
Tunning



Command Shaping



Command Shaping



Command Shaping

Initial Command

Input Shaper

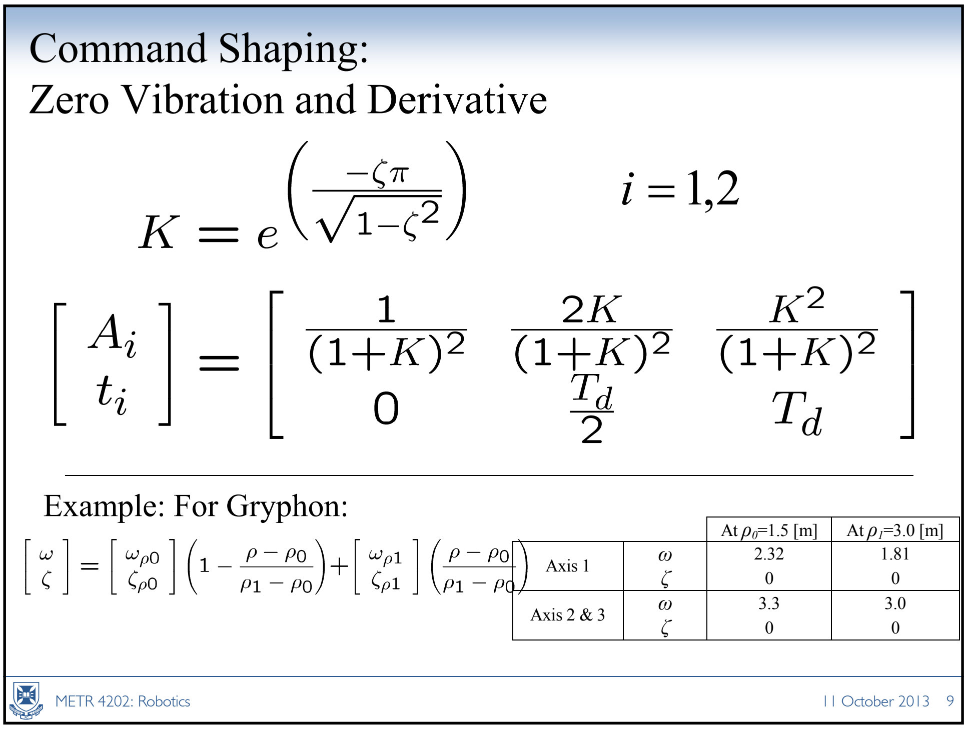
Shaped Command

- Zero Vibration (ZV)

$$\begin{bmatrix} A_i \\ t_i \end{bmatrix} = \begin{bmatrix} \dfrac{1}{1+K} & \dfrac{K}{1+K} \\ 0 & \dfrac{T_d}{2} \end{bmatrix} \qquad K = e^{\left(\frac{-\zeta\pi}{\sqrt{1-\zeta^2}}\right)}$$

- Zero Vibration and Derivative (ZVD)

$$\begin{bmatrix} A_i \\ t_i \end{bmatrix} = \begin{bmatrix} \dfrac{1}{(1+K)^2} & \dfrac{2K}{(1+K)^2} & \dfrac{K^2}{(1+K)^2} \\ 0 & \dfrac{T_d}{2} & T_d \end{bmatrix}$$



Command Shaping: Zero Vibration and Derivative

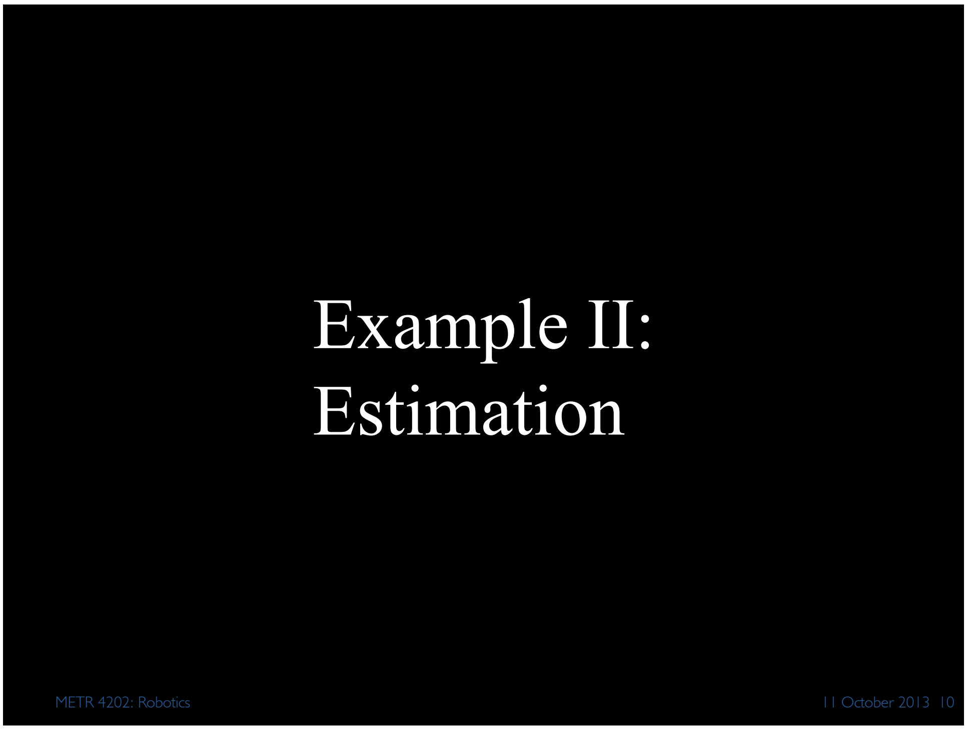
$$K = e^{\left(\frac{-\zeta\pi}{\sqrt{1-\zeta^2}}\right)} \qquad i = 1,2$$

$$\begin{bmatrix} A_i \\ t_i \end{bmatrix} = \begin{bmatrix} \frac{1}{(1+K)^2} & \frac{2K}{(1+K)^2} & \frac{K^2}{(1+K)^2} \\ 0 & \frac{T_d}{2} & T_d \end{bmatrix}$$

Example: For Gryphon:

$$\begin{bmatrix} \omega \\ \zeta \end{bmatrix} = \begin{bmatrix} \omega_{\rho 0} \\ \zeta_{\rho 0} \end{bmatrix} \left(1 - \frac{\rho - \rho_0}{\rho_1 - \rho_0}\right) + \begin{bmatrix} \omega_{\rho 1} \\ \zeta_{\rho 1} \end{bmatrix} \left(\frac{\rho - \rho_0}{\rho_1 - \rho_0}\right)$$

		At ρ_0=1.5 [m]	At ρ_1=3.0 [m]
Axis 1	ω	2.32	1.81
	ζ	0	0
Axis 2 & 3	ω	3.3	3.0
	ζ	0	0

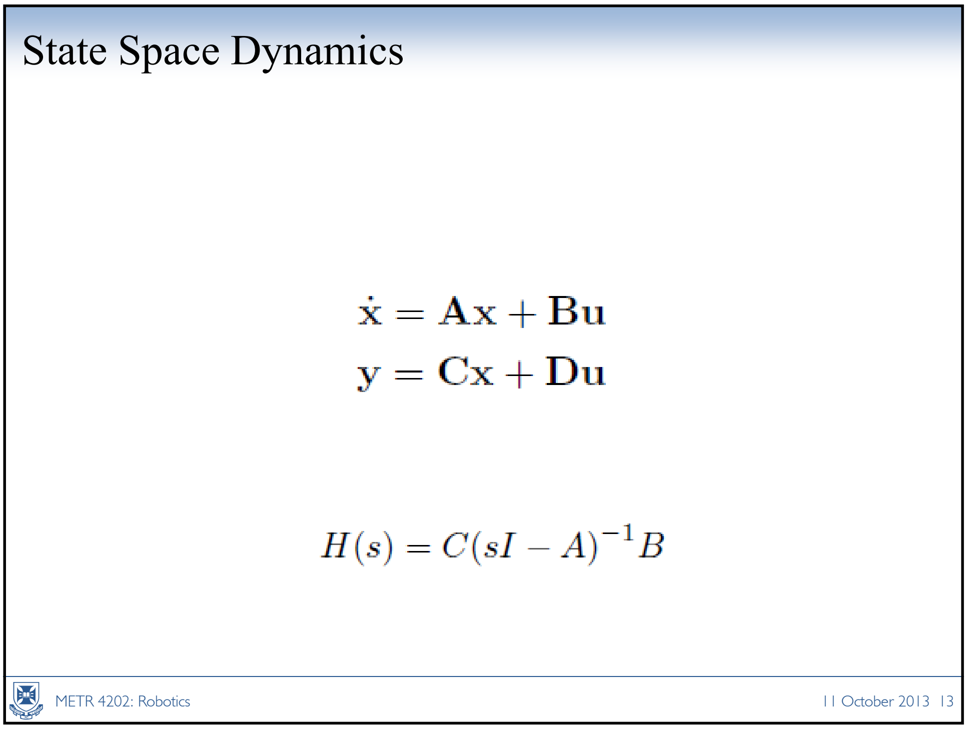


Example II: Estimation

Along multiple dimensions

State Space

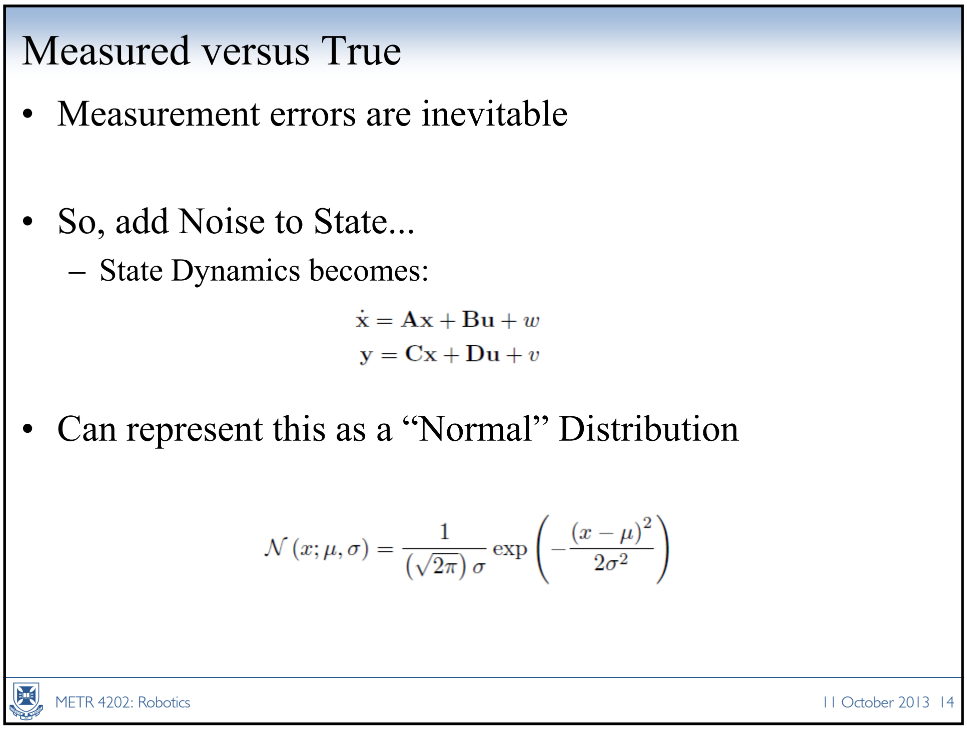
- We collect our set of uncertain variables into a vector …
  $\mathbf{x} = [x_1, x_2,\ldots, x_N]^T$
- The set of values that **x** might take on is termed the *state space*
- There is a *single* true value for **x**,
  but it is unknown



State Space Dynamics

$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{B}\mathbf{u}$$
$$\mathbf{y} = \mathbf{C}\mathbf{x} + \mathbf{D}\mathbf{u}$$

$$H(s) = C(sI - A)^{-1}B$$



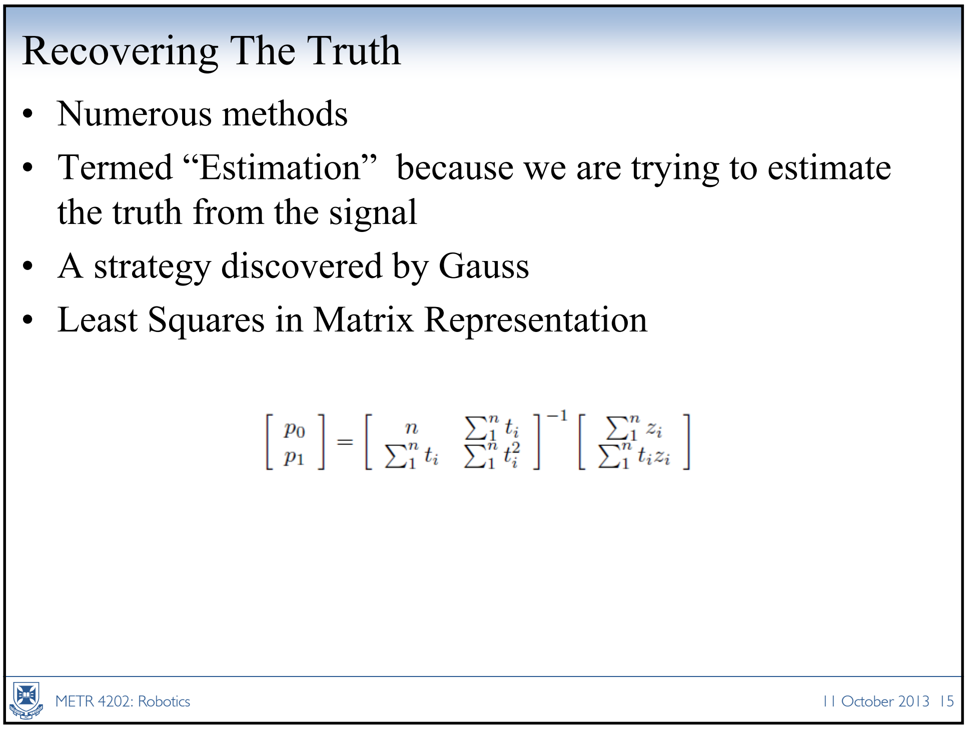
Measured versus True

- Measurement errors are inevitable
- So, add Noise to State...
  - State Dynamics becomes:

$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{B}\mathbf{u} + w$$
$$\mathbf{y} = \mathbf{C}\mathbf{x} + \mathbf{D}\mathbf{u} + v$$

- Can represent this as a “Normal” Distribution

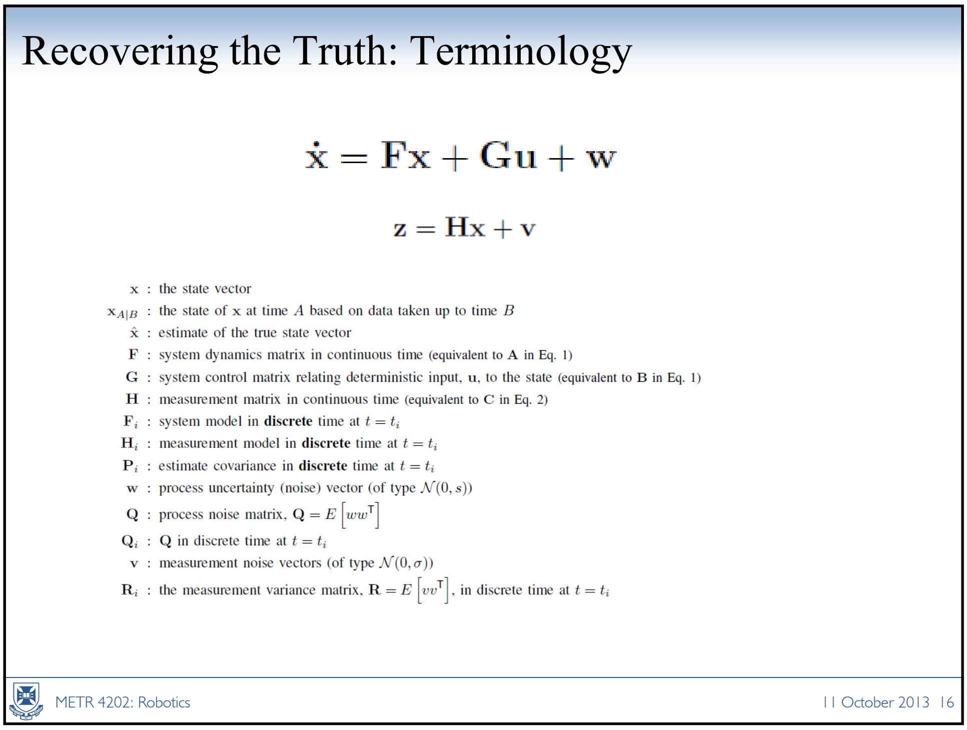
$$\mathcal{N}(x; \mu, \sigma) = \frac{1}{\left(\sqrt{2\pi}\right)\sigma} \exp\left(-\frac{(x-\mu)^2}{2\sigma^2}\right)$$



Recovering The Truth

- Numerous methods
- Termed “Estimation” because we are trying to estimate the truth from the signal
- A strategy discovered by Gauss
- Least Squares in Matrix Representation

$$\begin{bmatrix} p_0 \\ p_1 \end{bmatrix} = \begin{bmatrix} n & \sum_1^n t_i \\ \sum_1^n t_i & \sum_1^n t_i^2 \end{bmatrix}^{-1} \begin{bmatrix} \sum_1^n z_i \\ \sum_1^n t_i z_i \end{bmatrix}$$



Recovering the Truth: Terminology

$$\dot{\mathbf{x}} = \mathbf{F}\mathbf{x} + \mathbf{G}\mathbf{u} + \mathbf{w}$$

$$\mathbf{z} = \mathbf{H}\mathbf{x} + \mathbf{v}$$

$\mathbf{x}$: the state vector

$\mathbf{x}_{A|B}$: the state of $\mathbf{x}$ at time A based on data taken up to time B

$\hat{\mathbf{x}}$: estimate of the true state vector

$\mathbf{F}$: system dynamics matrix in continuous time (equivalent to $\mathbf{A}$ in Eq. 1)

$\mathbf{G}$: system control matrix relating deterministic input, $\mathbf{u}$, to the state (equivalent to $\mathbf{B}$ in Eq. 1)

$\mathbf{H}$: measurement matrix in continuous time (equivalent to $\mathbf{C}$ in Eq. 2)

$\mathbf{F}_i$: system model in **discrete** time at $t = t_i$

$\mathbf{H}_i$: measurement model in **discrete** time at $t = t_i$

$\mathbf{P}_i$: estimate covariance in **discrete** time at $t = t_i$

$\mathbf{w}$: process uncertainty (noise) vector (of type $\mathcal{N}(0, s)$)

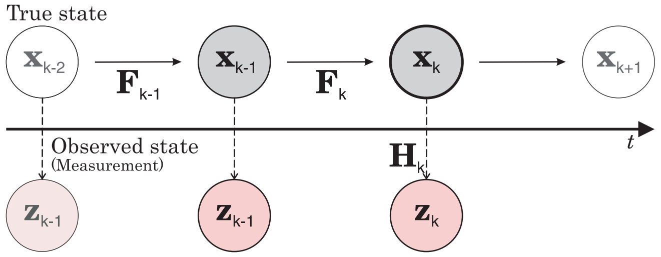
$\mathbf{Q}$: process noise matrix, $\mathbf{Q} = E\left[ww^{\mathsf{T}}\right]$

$\mathbf{Q}_i$: $\mathbf{Q}$ in discrete time at $t = t_i$

$\mathbf{v}$: measurement noise vectors (of type $\mathcal{N}(0, \sigma)$)

$\mathbf{R}_i$: the measurement variance matrix, $\mathbf{R} = E\left[vv^{\mathsf{T}}\right]$, in discrete time at $t = t_i$

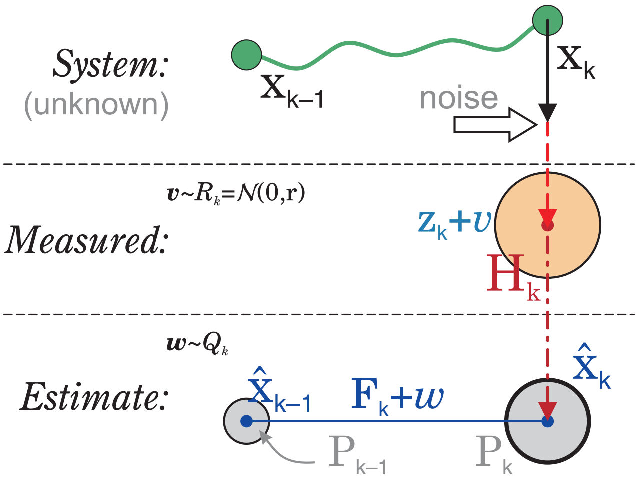
General Problem…



Duals and Dual Terminology

	Estimation		Control
Model:	$\dot{\mathbf{x}} = \mathbf{F}\mathbf{x}$ (discrete: $\mathbf{x} = \mathbf{F}_k\mathbf{x}$)	↔	$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x}$, $\mathbf{A} = \mathbf{F}^{\top}$
Regulates:	P (covariance)	↔	M (performance matrix)
Minimized function:	Q (or $GQG^{\top}$)	↔	V
Optimal Gain:	K	↔	G
Completeness law:	Observability	↔	Controllability

Estimation Process in Pictures

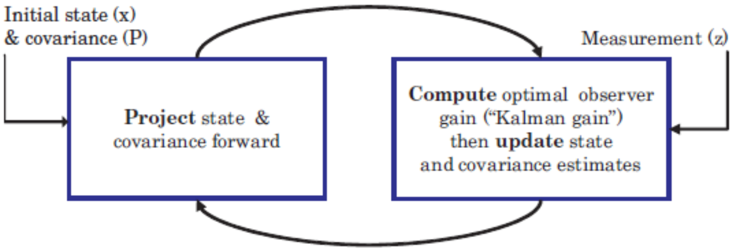

System:

(unknown)

Measured:

Estimate:

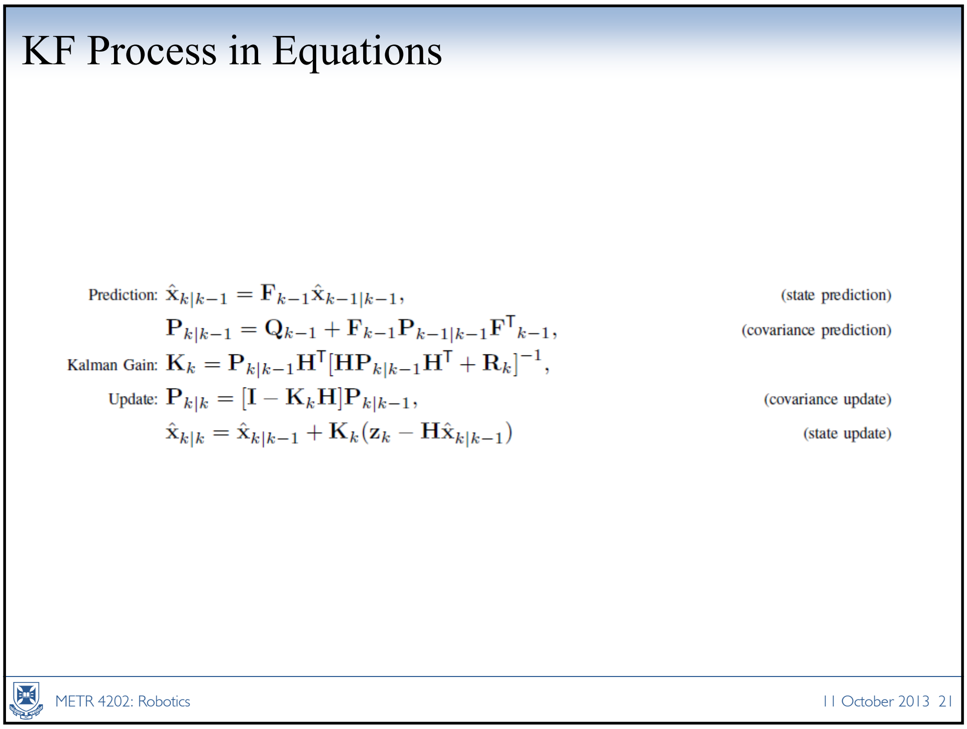
Kalman Filter Process


Initial state (x)
& covariance (P)

Project state &
covariance forward

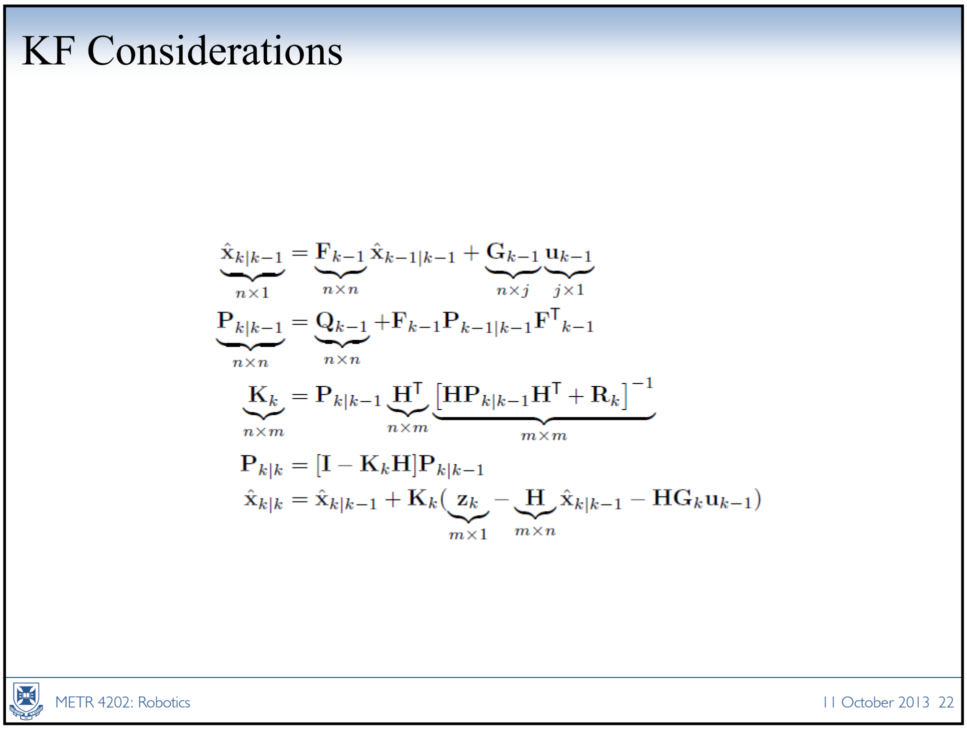
Compute optimal observer
gain ("Kalman gain")
then **update** state
and covariance estimates

Measurement (z)



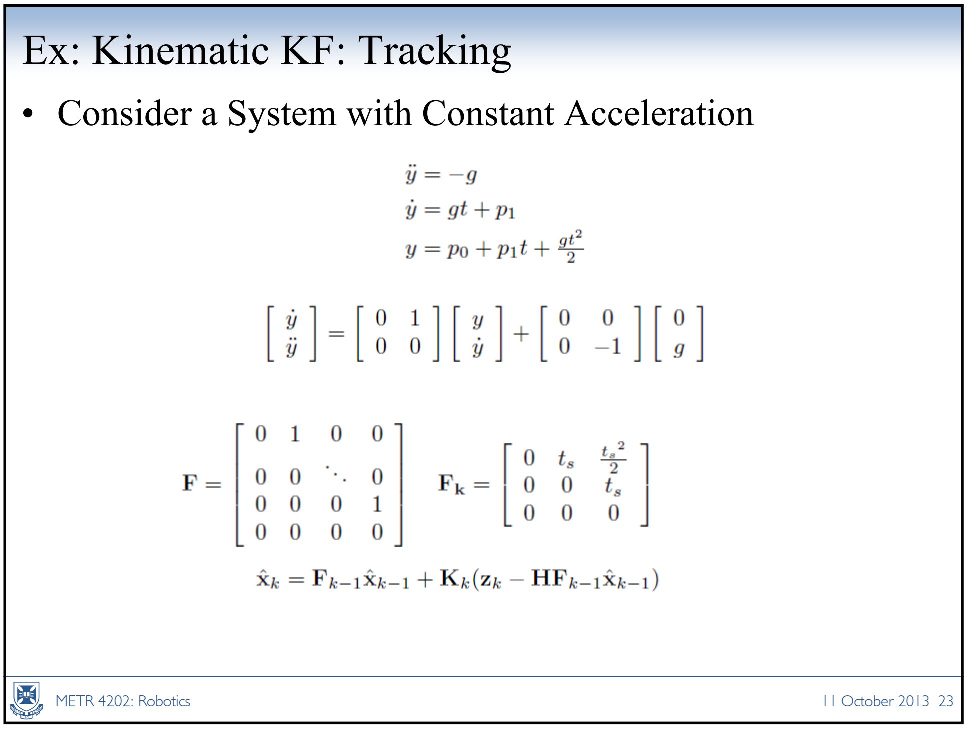
KF Process in Equations

$$\begin{aligned}
\text{Prediction: } \hat{\mathbf{x}}_{k|k-1} &= \mathbf{F}_{k-1}\hat{\mathbf{x}}_{k-1|k-1}, && \text{(state prediction)}\\
\mathbf{P}_{k|k-1} &= \mathbf{Q}_{k-1} + \mathbf{F}_{k-1}\mathbf{P}_{k-1|k-1}\mathbf{F}^{\mathsf{T}}{}_{k-1}, && \text{(covariance prediction)}\\
\text{Kalman Gain: } \mathbf{K}_k &= \mathbf{P}_{k|k-1}\mathbf{H}^{\mathsf{T}}[\mathbf{H}\mathbf{P}_{k|k-1}\mathbf{H}^{\mathsf{T}} + \mathbf{R}_k]^{-1}, &&\\
\text{Update: } \mathbf{P}_{k|k} &= [\mathbf{I} - \mathbf{K}_k\mathbf{H}]\mathbf{P}_{k|k-1}, && \text{(covariance update)}\\
\hat{\mathbf{x}}_{k|k} &= \hat{\mathbf{x}}_{k|k-1} + \mathbf{K}_k(\mathbf{z}_k - \mathbf{H}\hat{\mathbf{x}}_{k|k-1}) && \text{(state update)}
\end{aligned}$$



KF Considerations

$$\begin{aligned}
\underbrace{\hat{\mathbf{x}}_{k|k-1}}_{n\times 1} &= \underbrace{\mathbf{F}_{k-1}}_{n\times n}\hat{\mathbf{x}}_{k-1|k-1} + \underbrace{\mathbf{G}_{k-1}}_{n\times j}\underbrace{\mathbf{u}_{k-1}}_{j\times 1}\\
\underbrace{\mathbf{P}_{k|k-1}}_{n\times n} &= \underbrace{\mathbf{Q}_{k-1}}_{n\times n} + \mathbf{F}_{k-1}\mathbf{P}_{k-1|k-1}\mathbf{F}^{\mathsf{T}}{}_{k-1}\\
\underbrace{\mathbf{K}_k}_{n\times m} &= \mathbf{P}_{k|k-1}\underbrace{\mathbf{H}^{\mathsf{T}}}_{n\times m}\underbrace{[\mathbf{H}\mathbf{P}_{k|k-1}\mathbf{H}^{\mathsf{T}} + \mathbf{R}_k]^{-1}}_{m\times m}\\
\mathbf{P}_{k|k} &= [\mathbf{I} - \mathbf{K}_k\mathbf{H}]\mathbf{P}_{k|k-1}\\
\hat{\mathbf{x}}_{k|k} &= \hat{\mathbf{x}}_{k|k-1} + \mathbf{K}_k(\underbrace{\mathbf{z}_k}_{m\times 1} - \underbrace{\mathbf{H}}_{m\times n}\hat{\mathbf{x}}_{k|k-1} - \mathbf{H}\mathbf{G}_k\mathbf{u}_{k-1})
\end{aligned}$$



Ex: Kinematic KF: Tracking

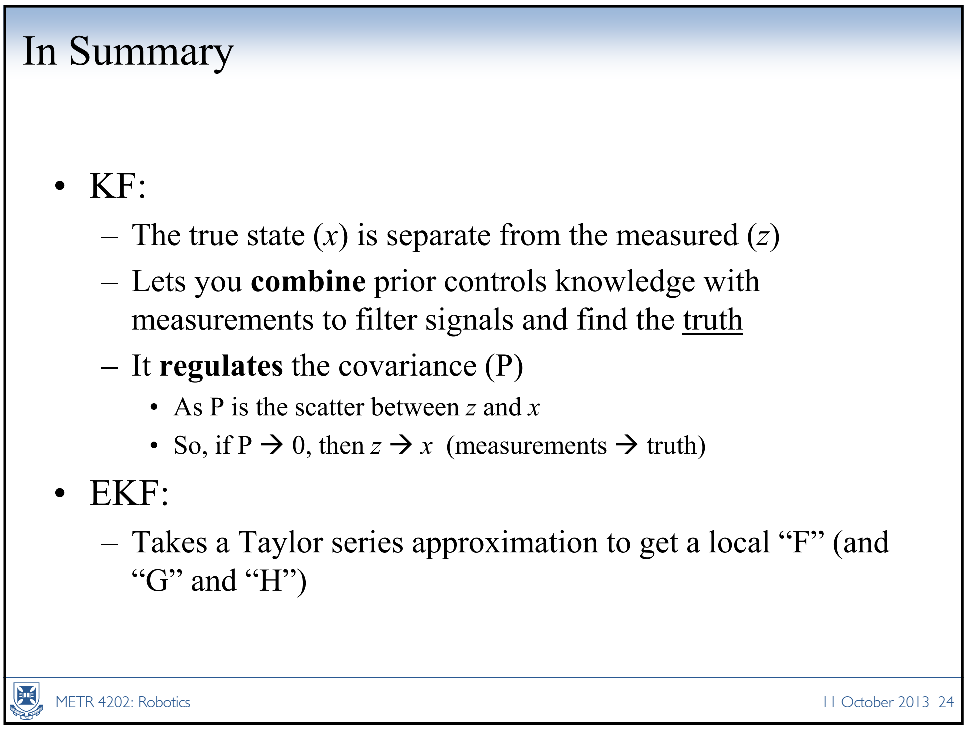
- Consider a System with Constant Acceleration

$$\ddot{y} = -g$$
$$\dot{y} = gt + p_1$$
$$y = p_0 + p_1 t + \frac{gt^2}{2}$$

$$\begin{bmatrix} \dot{y} \\ \ddot{y} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} y \\ \dot{y} \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} 0 \\ g \end{bmatrix}$$

$$\mathbf{F} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & \ddots & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \quad \mathbf{F_k} = \begin{bmatrix} 0 & t_s & \frac{{t_s}^2}{2} \\ 0 & 0 & t_s \\ 0 & 0 & 0 \end{bmatrix}$$

$$\hat{\mathbf{x}}_k = \mathbf{F}_{k-1}\hat{\mathbf{x}}_{k-1} + \mathbf{K}_k(\mathbf{z}_k - \mathbf{H}\mathbf{F}_{k-1}\hat{\mathbf{x}}_{k-1})$$

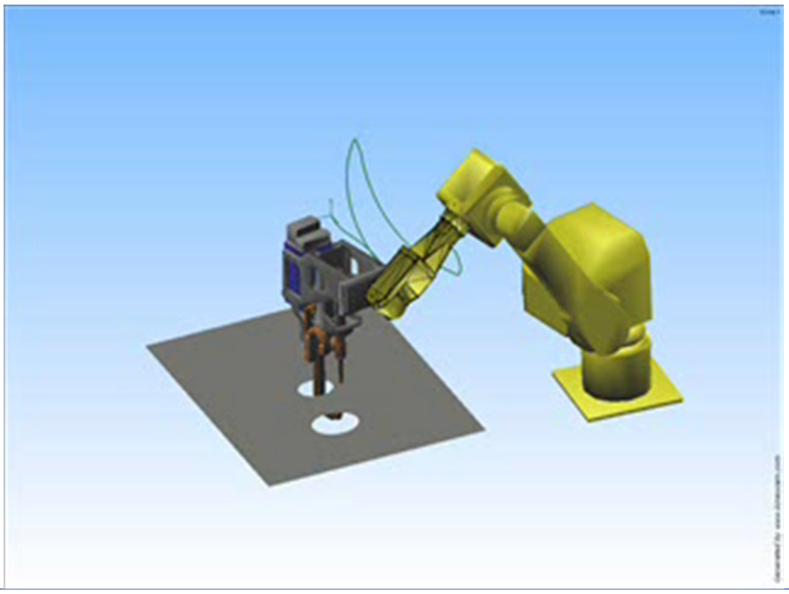


In Summary

- KF:
  - The true state (*x*) is separate from the measured (*z*)
  - Lets you **combine** prior controls knowledge with measurements to filter signals and find the truth
  - It **regulates** the covariance (P)
    - As P is the scatter between *z* and *x*
    - So, if P → 0, then *z* → *x* (measurements → truth)
- EKF:
  - Takes a Taylor series approximation to get a local “F” (and “G” and “H”)

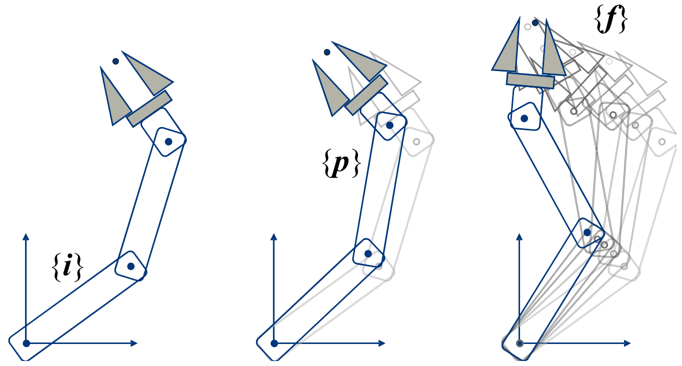
Motion
PLANNING

Trajectory Generation & Planning



Trajectory Generation

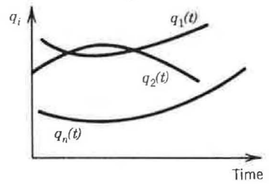
- The goal is to get from an initial position $\{\boldsymbol{i}\}$ to a final position $\{\boldsymbol{f}\}$ via a path points $\{\boldsymbol{p}\}$



Joint Space

Consider only the **joint positions** as a function of time

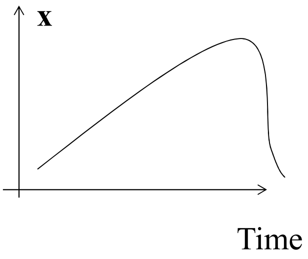
- + Since we control the joints, this is more direct
- -- If we want to follow a particular trajectory, <u>not easy</u>
  - – at best lots of intermediate points
  - – No guarantee that you can solve the Inverse Kinematics for all path points



Cartesian Workspace

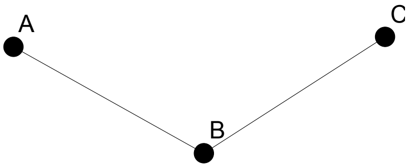
Consider the **Cartesian positions** as a function of time

- + Can track shapes exactly
- -- We need to solve the inverse kinematics and dynamics



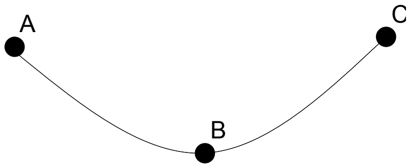
Polynomial Trajectories

- Straight line Trajectories



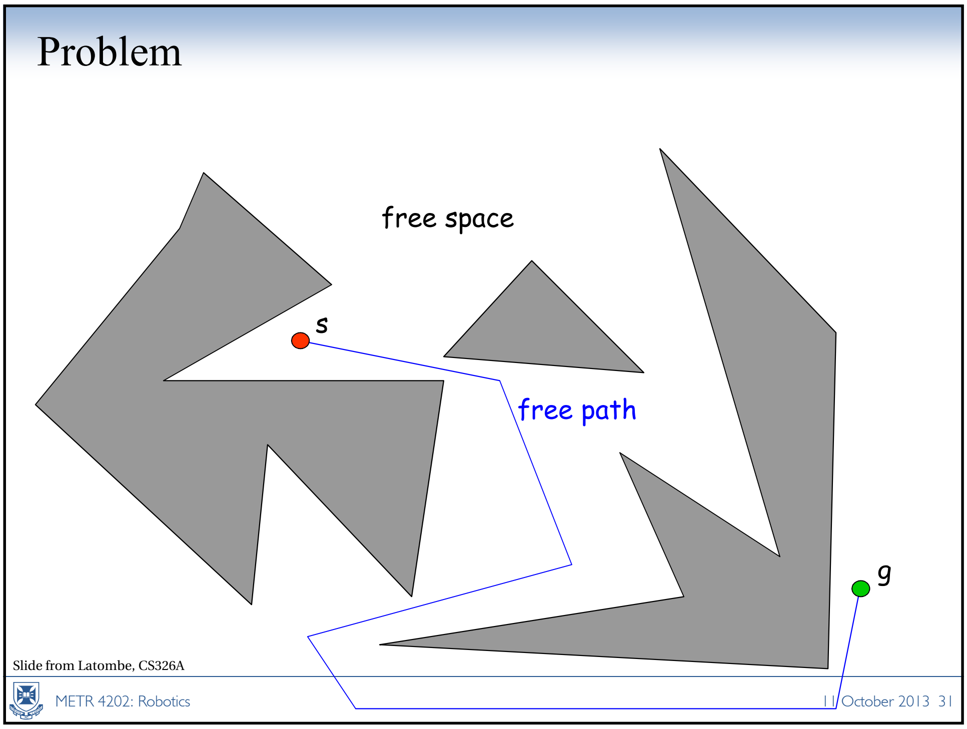
- Simpler

- Polynomial Trajectories



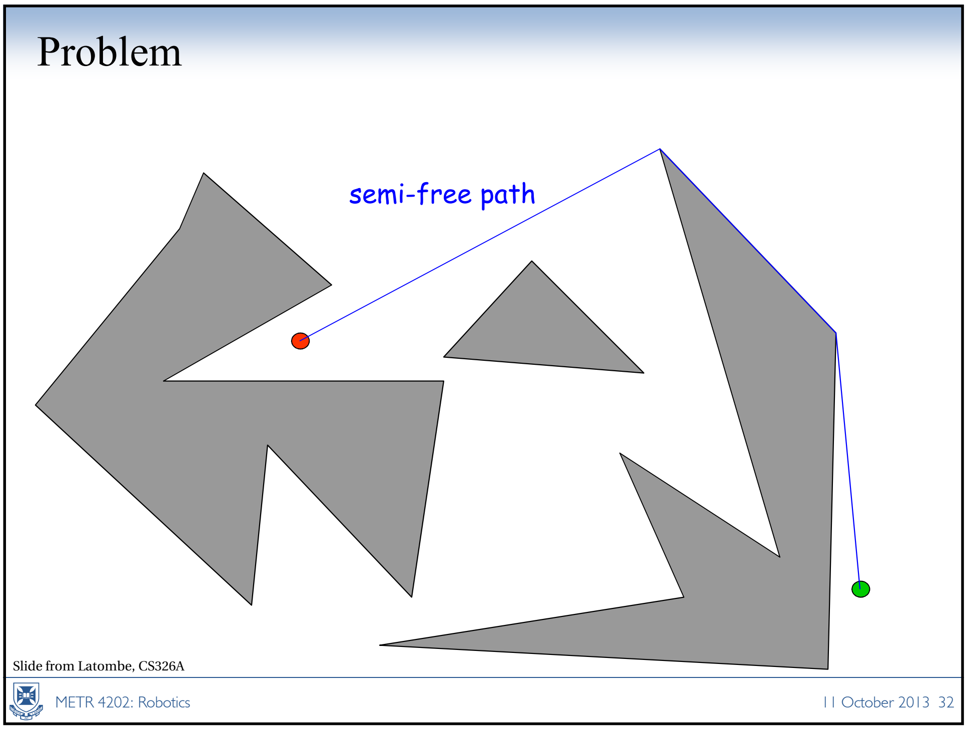
$u(t) = a_0 + a_1 t + a_2 t^2 + a_3 t^3$

- Parabolic blends are smoother
- Use “pseudo via points”



Problem

Slide from Latombe, CS326A



Problem

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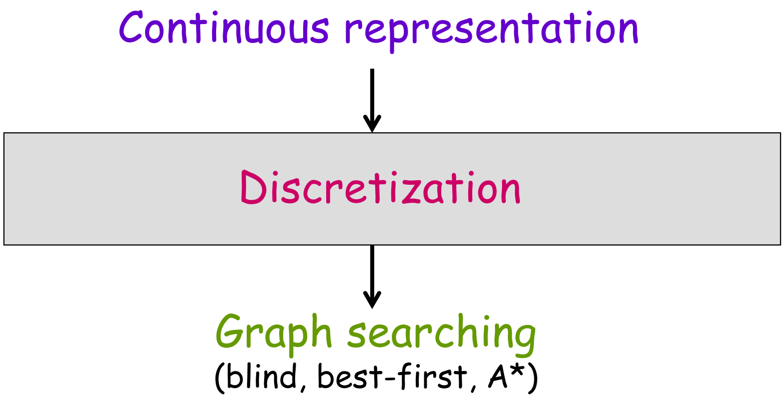
Motion-Planning Framework



Continuous representation

Discretization

Graph searching

(blind, best-first, A*)

Slide from Latombe, CS326A

Path-Planning Approaches

- Roadmap
  Represent the connectivity of the free space by a network of 1-D curves
- Cell decomposition
  Decompose the free space into simple cells and represent the connectivity of the free space by the adjacency graph of these cells
- Potential field
  Define a function over the free space that has a global minimum at the goal configuration and follow its steepest descent

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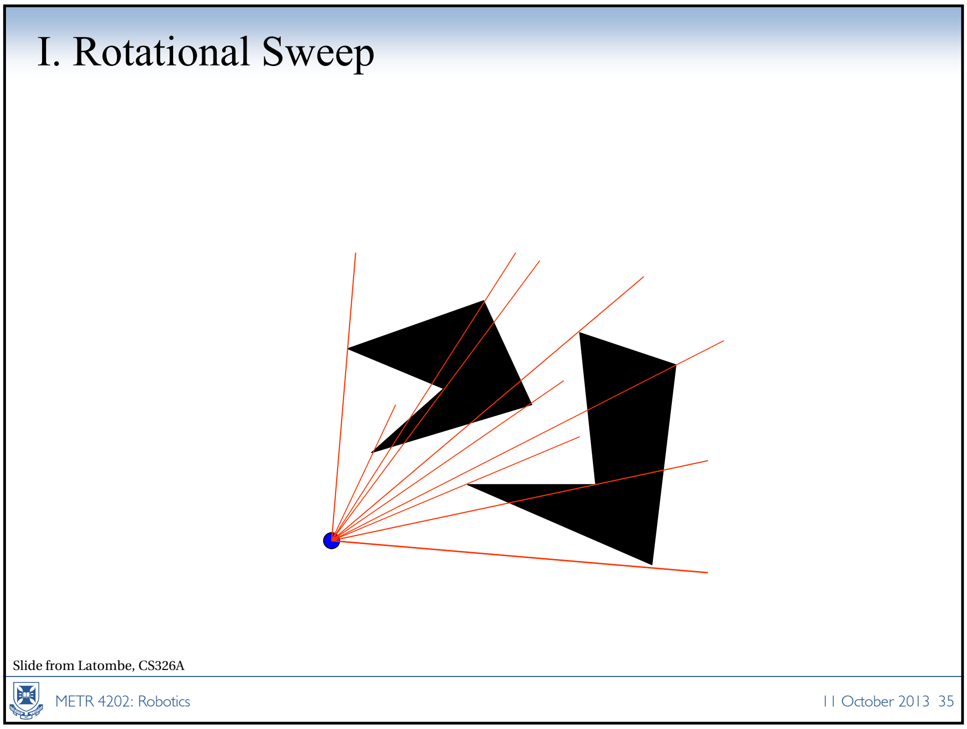

I. Rotational Sweep

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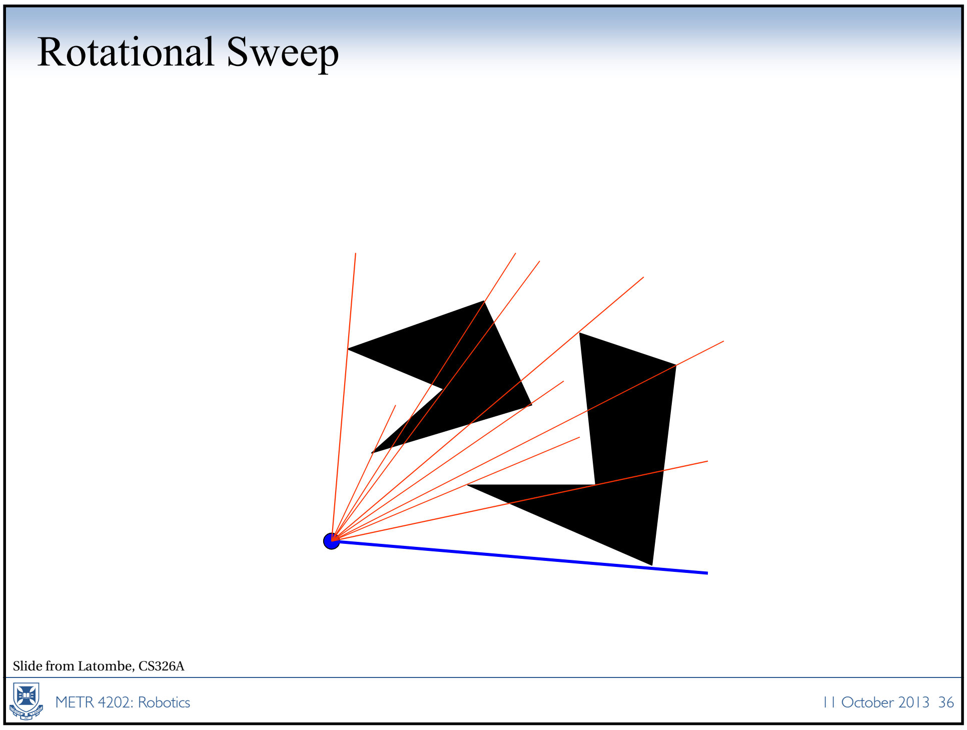

Rotational Sweep

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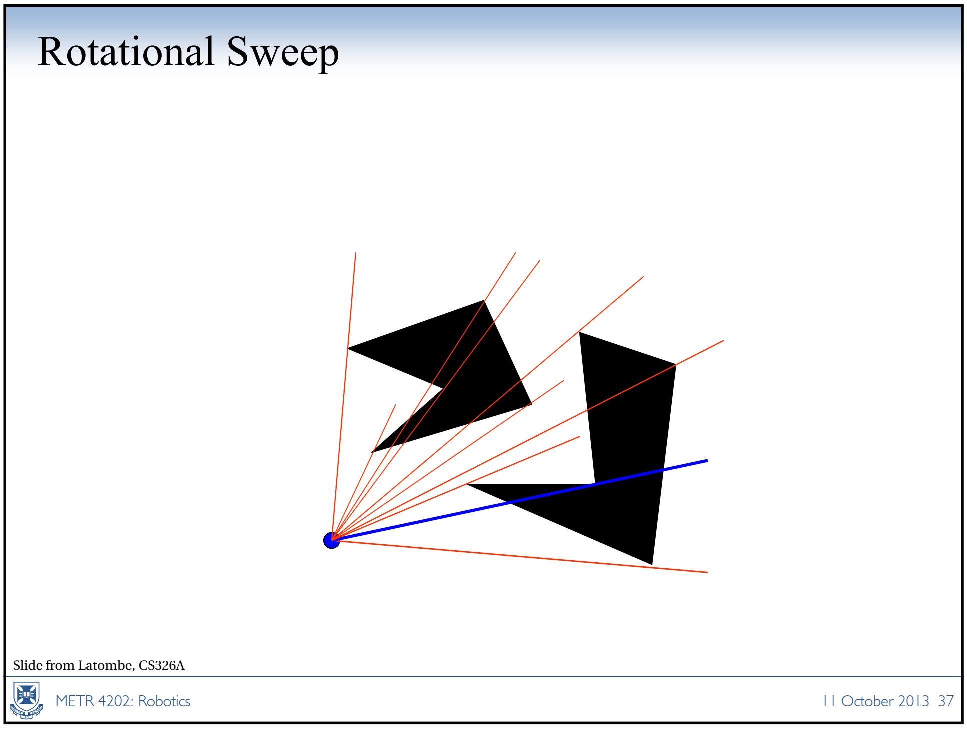

Rotational Sweep

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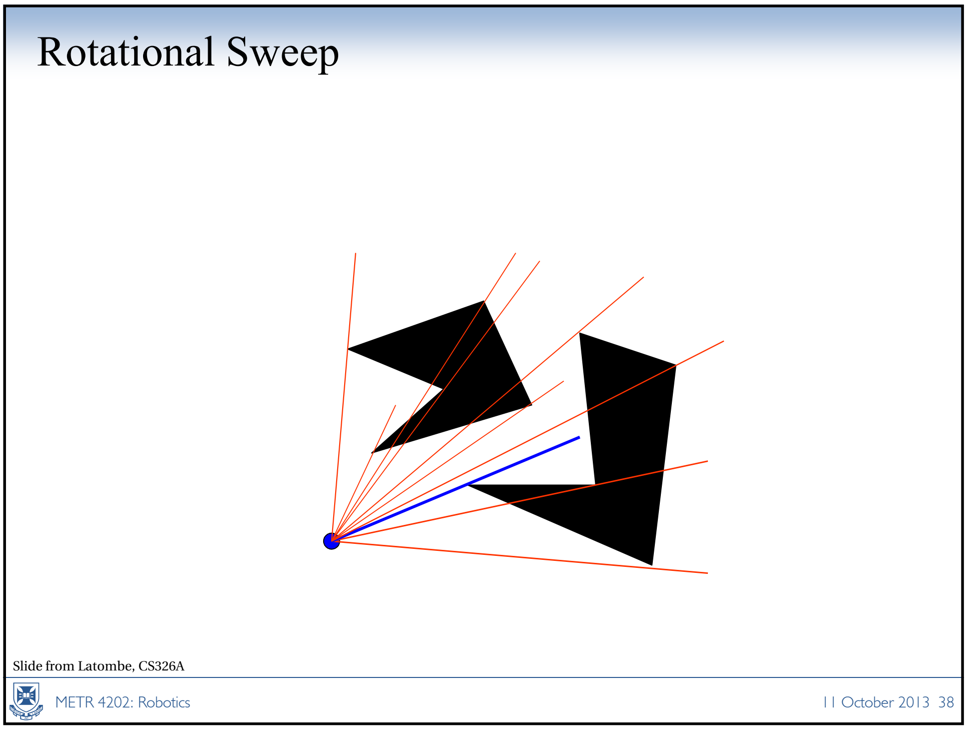

Rotational Sweep

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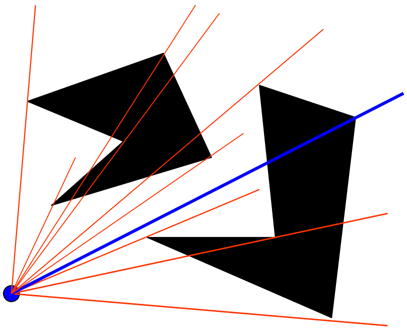
Rotational Sweep



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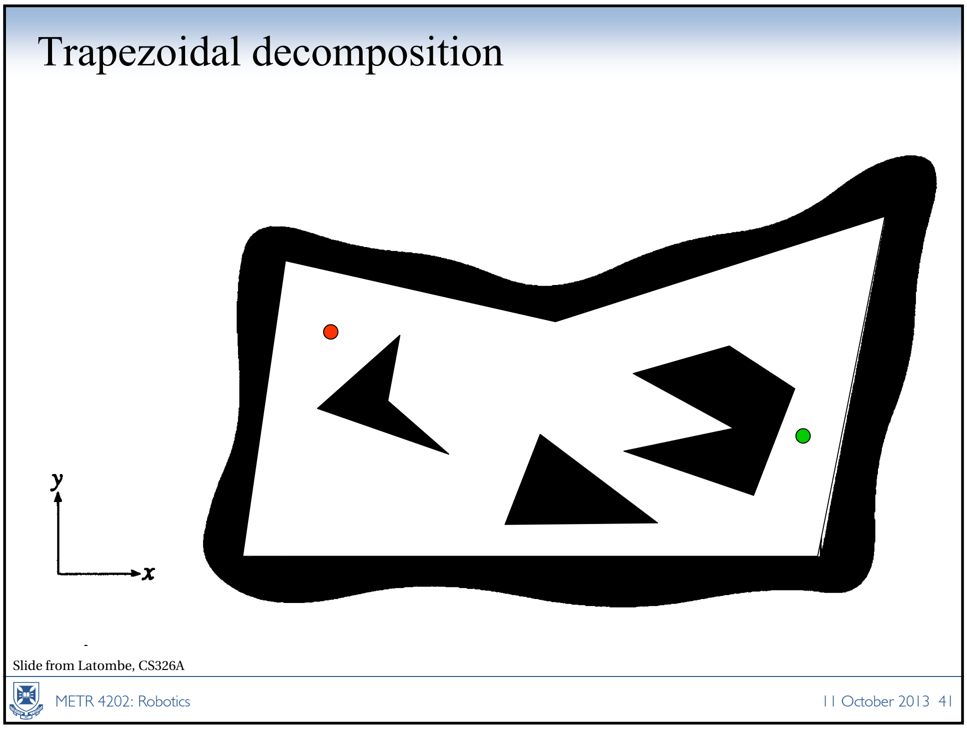
II. Cell-Decomposition Methods

Two classes of methods:

- Exact cell decomposition
  - The free space **F** is represented by a collection of non-overlapping cells whose union is exactly **F**
  - Example: trapezoidal decomposition
- Approximate cell decomposition
  - **F** is represented by a collection of non-overlapping cells whose union is contained in **F** Examples: quadtree, octree, 2n-tree



Trapezoidal decomposition

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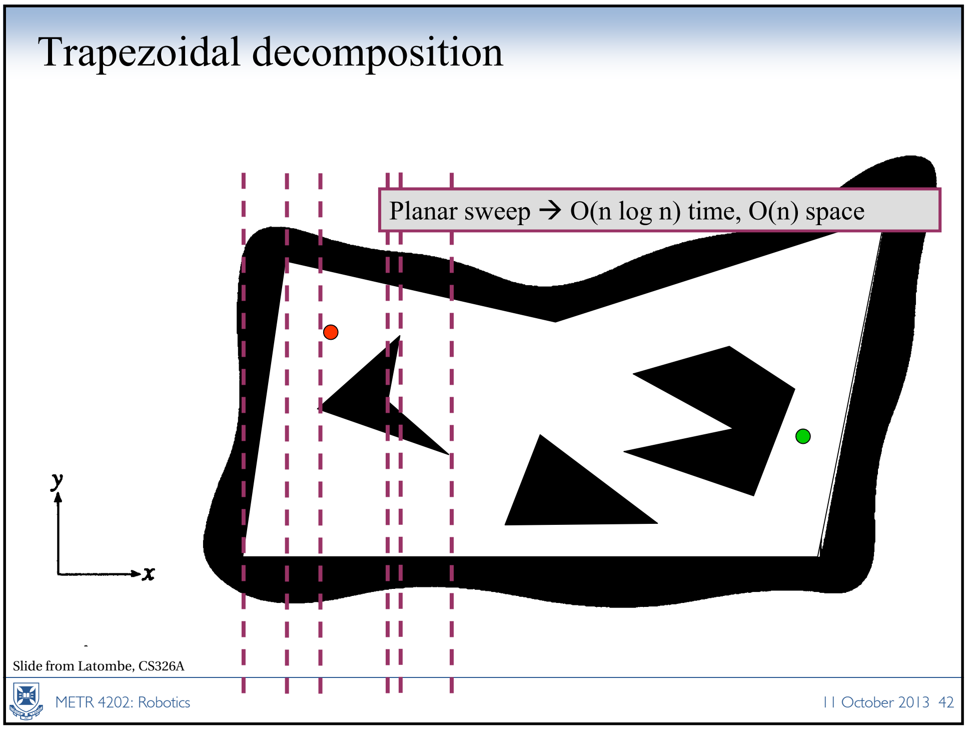

Trapezoidal decomposition

Planar sweep → O(n log n) time, O(n) space

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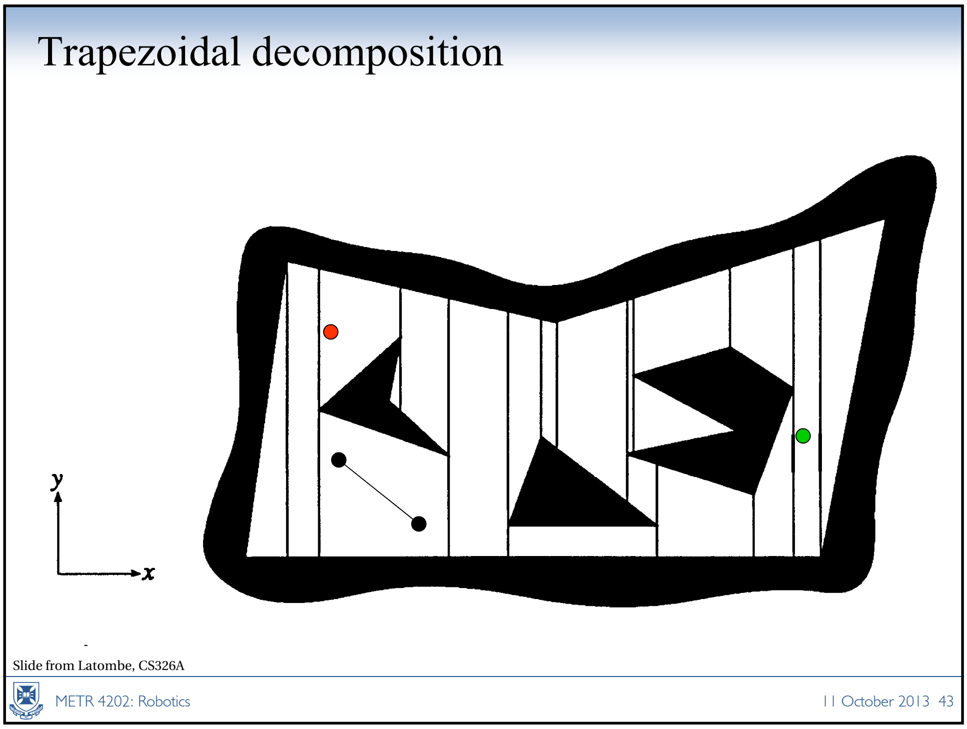

Trapezoidal decomposition

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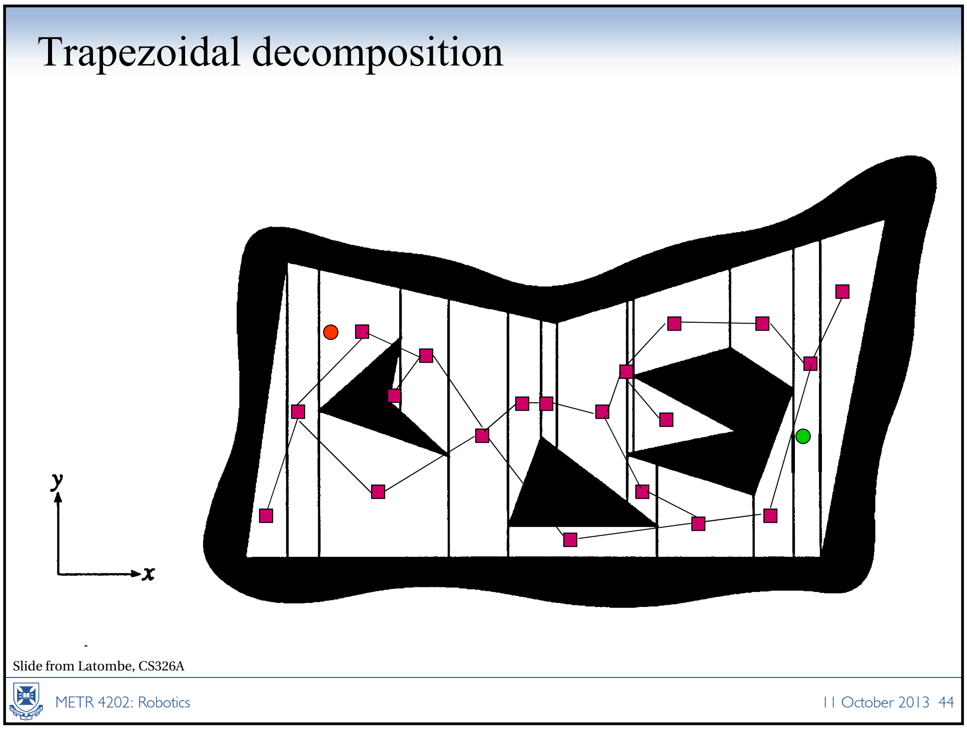

Trapezoidal decomposition

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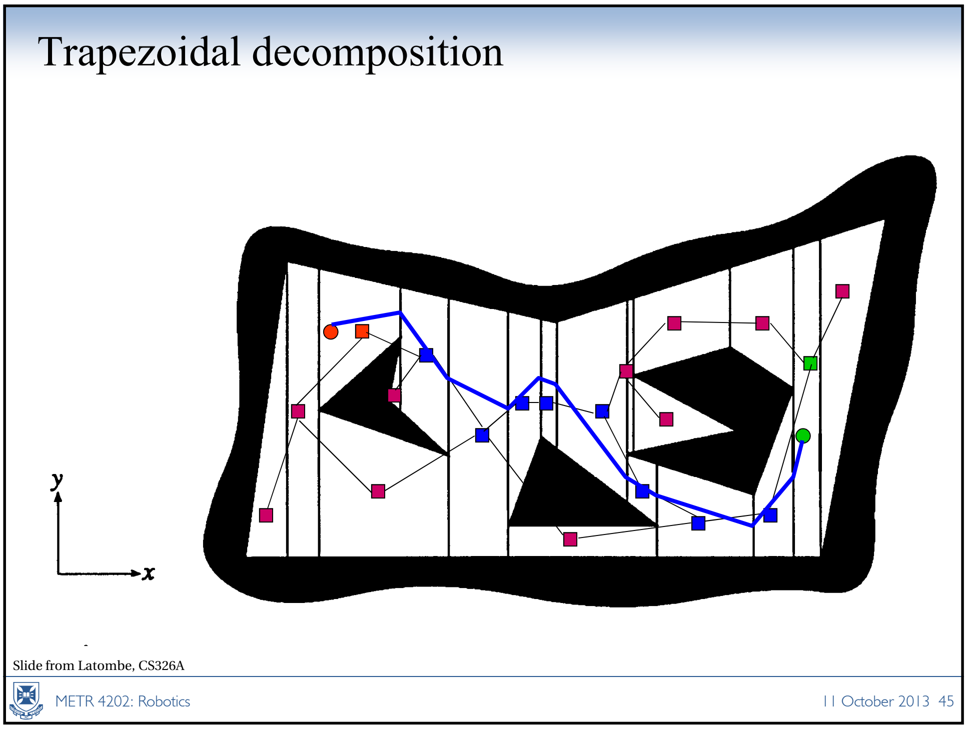

Trapezoidal decomposition

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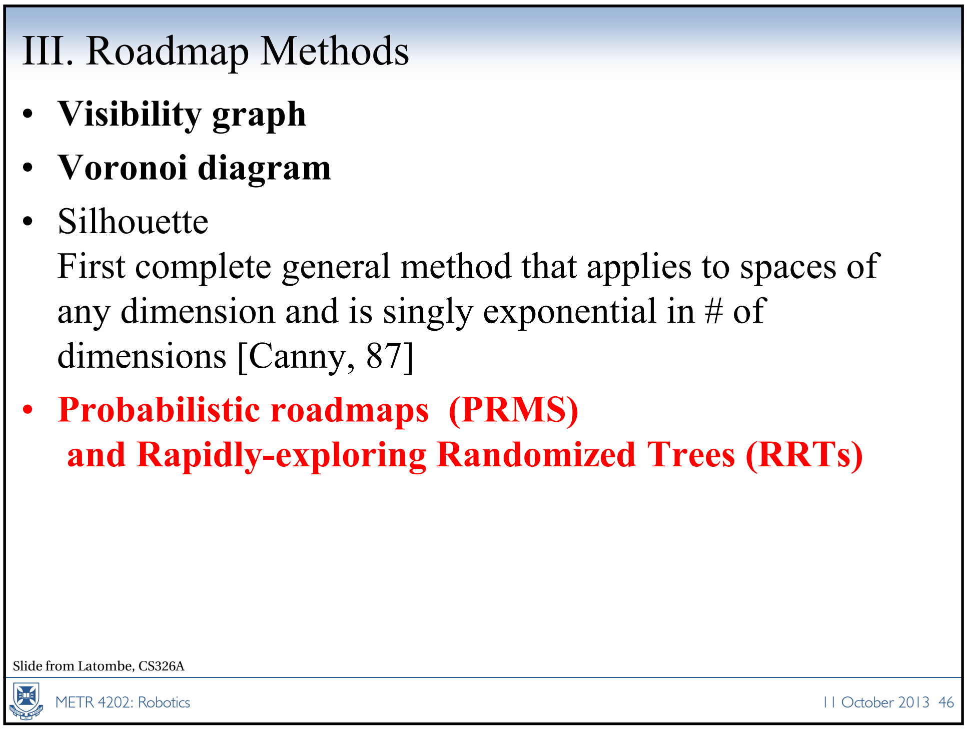

III. Roadmap Methods

- **Visibility graph**
- **Voronoi diagram**
- Silhouette
  First complete general method that applies to spaces of any dimension and is singly exponential in # of dimensions [Canny, 87]
- **Probabilistic roadmaps (PRMS)
  and Rapidly-exploring Randomized Trees (RRTs)**

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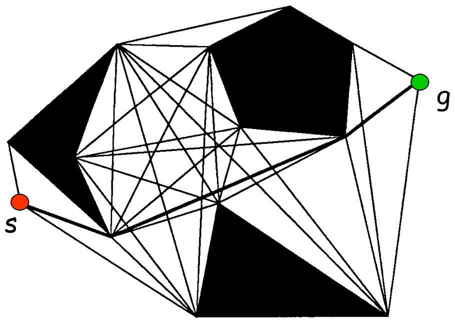
Roadmap Methods

- **Visibility graph**
  Introduced in the Shakey project at SRI in the late 60s.
  Can produce shortest paths in 2-D configuration spaces

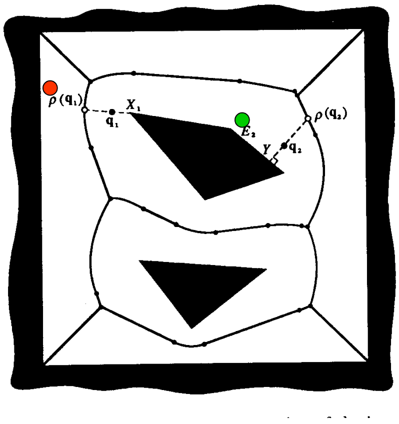


Slide from Latombe, CS326A

Roadmap Methods

- Voronoi diagram
  Introduced by Computational Geometry researchers. Generate paths that maximizes clearance.

  O(n log n) time
  O(n) space



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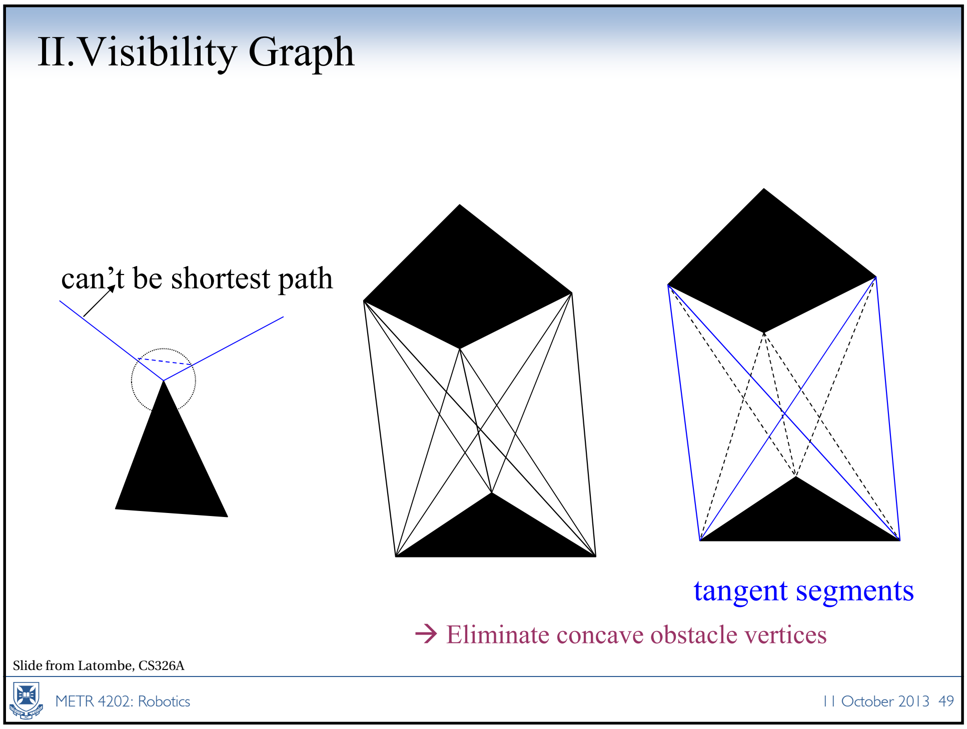

II.Visibility Graph

can't be shortest path

tangent segments

→ Eliminate concave obstacle vertices

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METR 4202: Robotics

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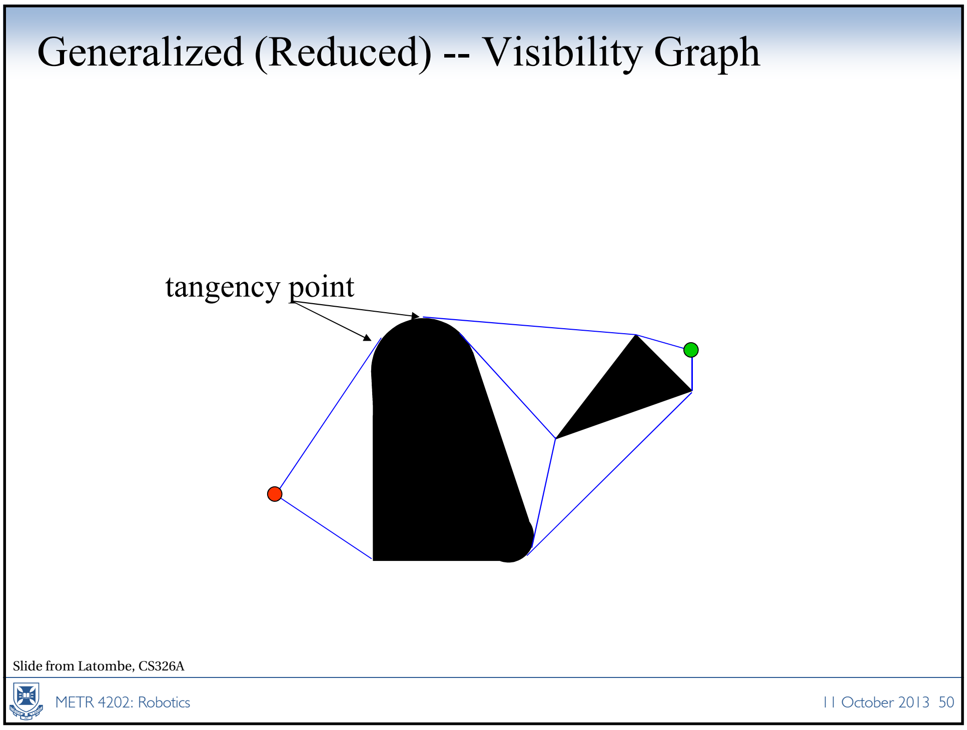

Generalized (Reduced) -- Visibility Graph

tangency point

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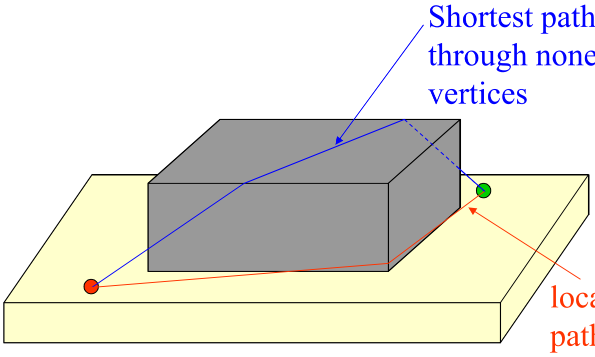
Three-Dimensional Space



Shortest path passes through none of the vertices

locally shortest path homotopic to globally shortest path

Computing the shortest collision-free path in a polyhedral space is NP-hard

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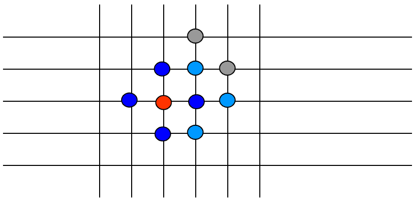
Sketch of Grid Algorithm (with best-first search)

- Place regular grid G over space
- Search G using best-first search algorithm with potential as heuristic function



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Simple Algorithm (for Visibility Graphs)

- Install all obstacles vertices in VG, plus the start and goal positions
- For every pair of nodes u, v in VG

```
    If segment(u,v) is an obstacle edge then
            insert (u,v) into VG
    else
    for every obstacle edge e
            if segment(u,v) intersects e
                    then go up to segment
  insert (u,v) into VG
```

- Search VG using A*

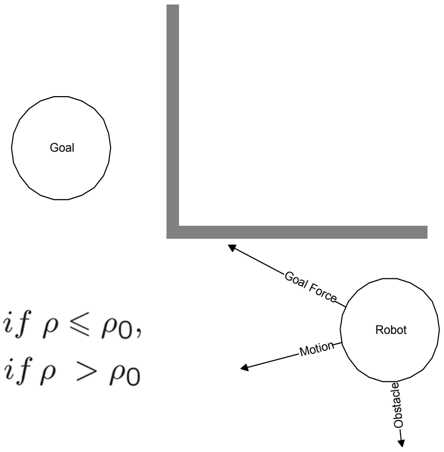
Slide based on Latombe, CS326A

IV. Potential Field Methods

- Approach initially proposed for real-time collision avoidance [Khatib, 86]

$$F_{Goal} = -k_p(x - x_{Goal})$$

$$F_{Obstacle} = \begin{cases} \eta \left(\frac{1}{\rho} - \frac{1}{\rho_0}\right) \frac{1}{\rho^2} \frac{\partial \rho}{\partial x} & if\ \rho \leqslant \rho_0, \\ 0 & if\ \rho\ > \rho_0 \end{cases}$$



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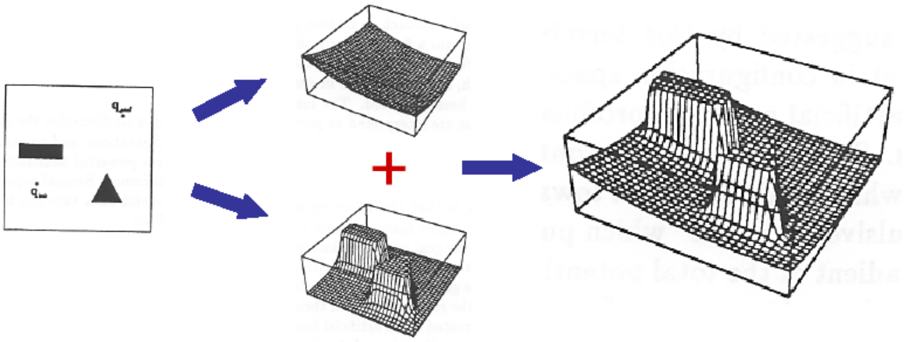
Attractive and Repulsive fields



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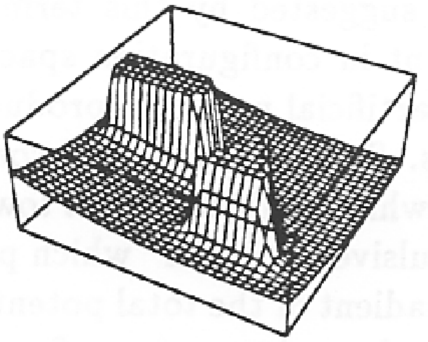
Local-Minimum Issue



- Perform best-first search (possibility of combining with approximate cell decomposition)
- Alternate descents and random walks
- Use local-minimum-free potential (navigation function)

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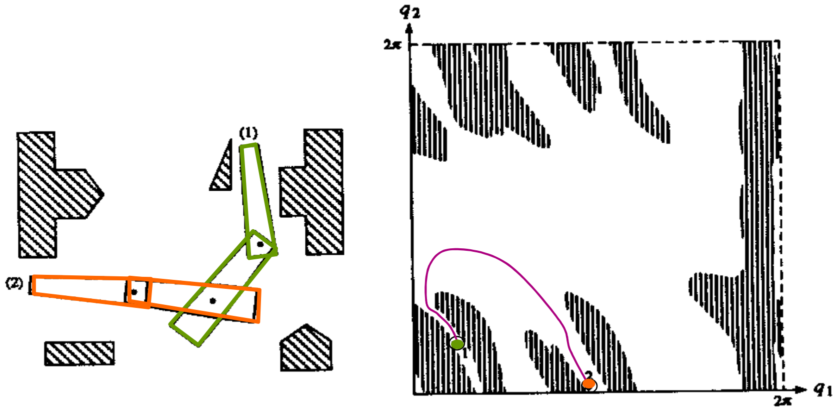
Configuration Space



- A robot configuration is a specification of the positions of all robot points relative to a fixed coordinate system
- Usually a configuration is expressed as a “vector” of position/orientation parameters

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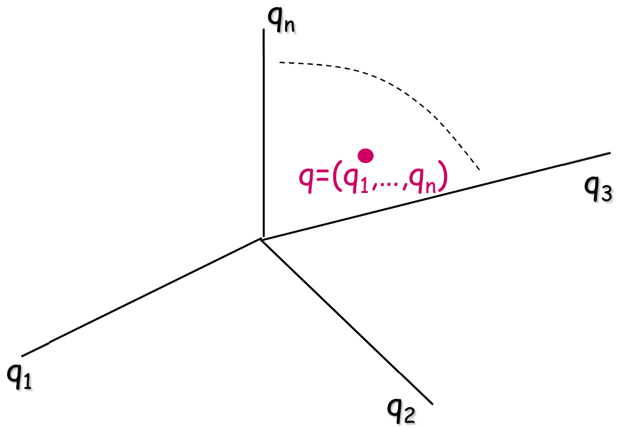
Motion Planning in C-Space



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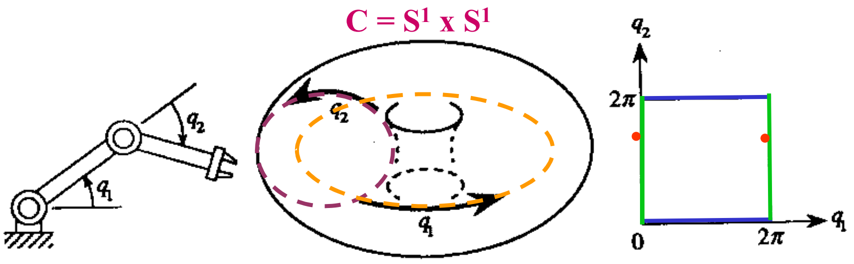
Configuration Space of a Robot

- Space of all its possible configurations
- But the topology of this space is usually not that of a Cartesian space



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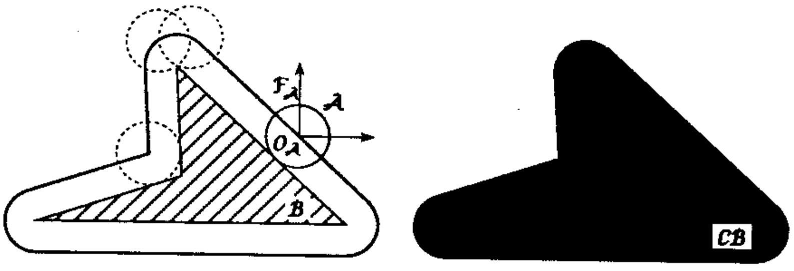
Disc Robot in 2-D Workspace



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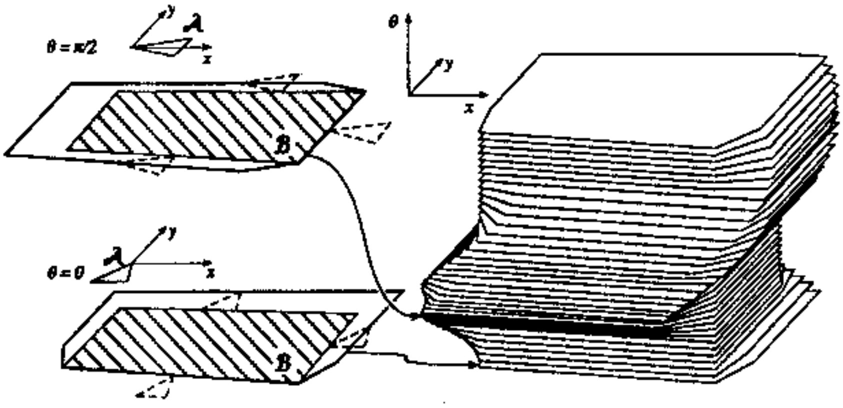
Rigid Robot Translating and Rotating in 2-D



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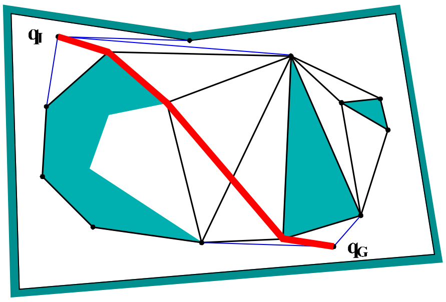
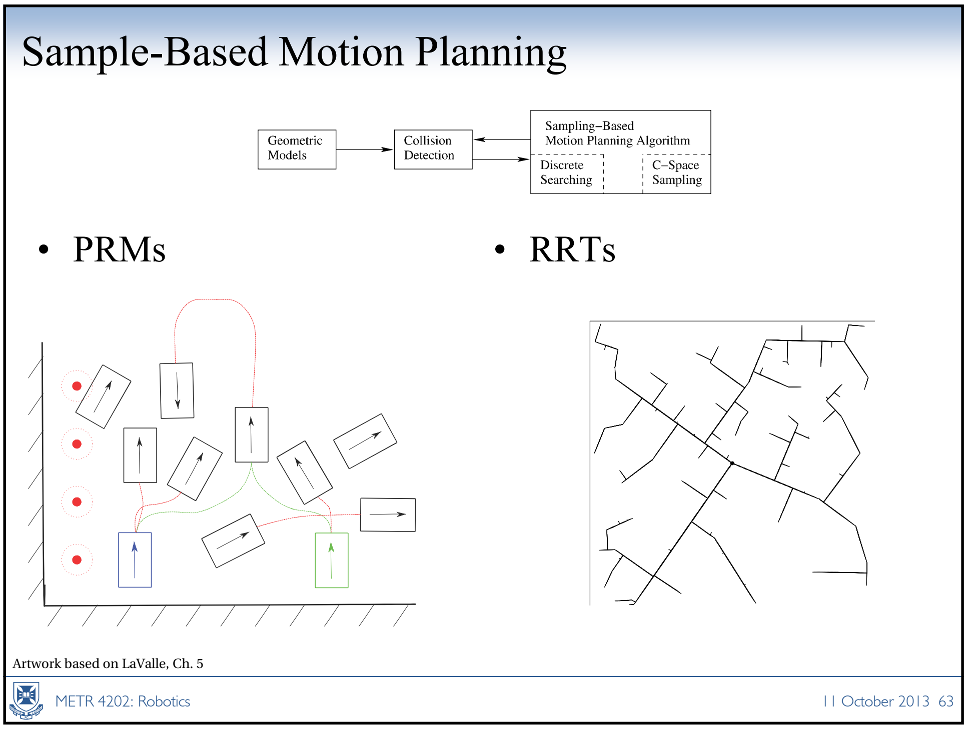
Geometric Planning Methods



- Several Geometric Methods:
  - Vertical (Trapezoidal) Cell Decomposition
  - **Roadmap Methods**
    - **Cell (Triangular) Decomposition**
    - Visibility Graphs
    - Veroni Graphs

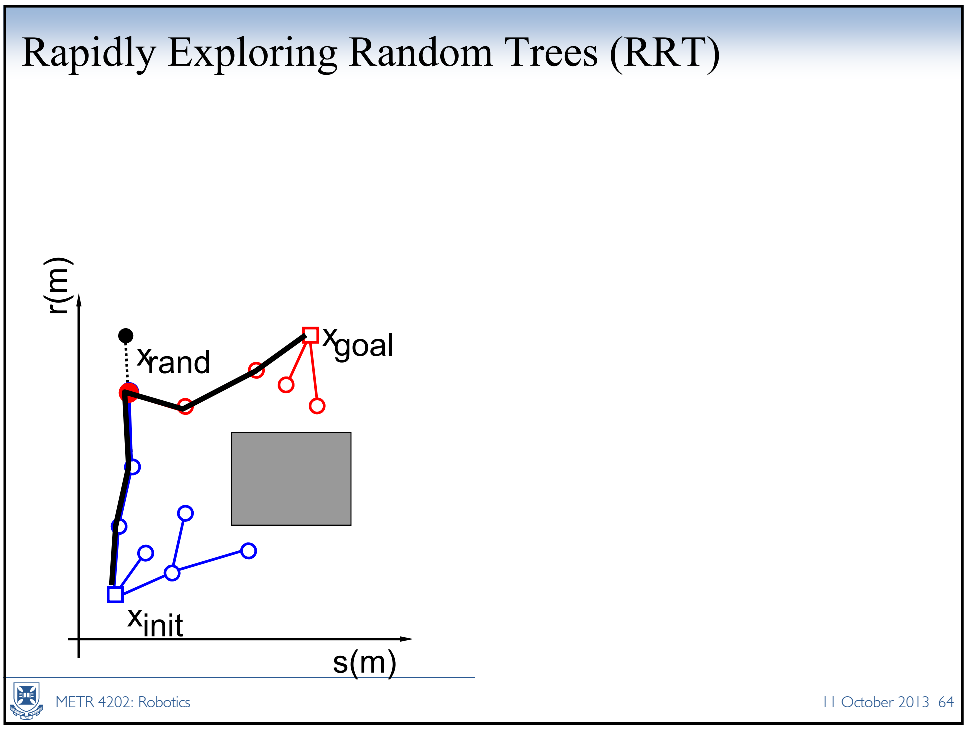
Artwork from LaValle, Ch. 6



Sample-Based Motion Planning

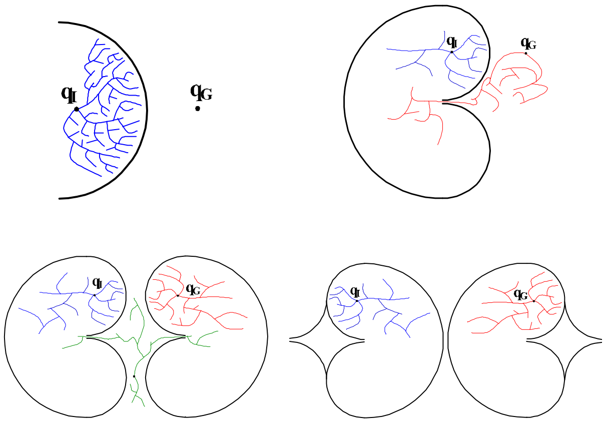
- PRMs
- RRTs

Artwork based on LaValle, Ch. 5



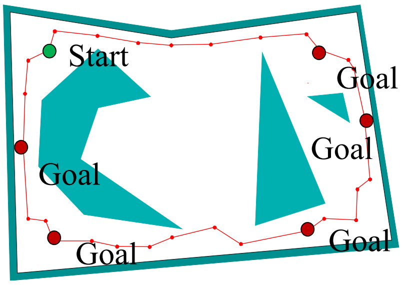
Rapidly Exploring Random Trees (RRT)

Sampling and the "Bug Trap" Problem



Artwork based on LaValle, Ch. 5

Multiple Points & Sequencing



- Sequencing
  - Determining the "best" order to go in

➔ Travelling Salesman Problem

A salesman has to visit each city on a given list exactly once. In doing this, he **starts** from his home city and in the **end he has to return to his home** city. It is plausible for him to select the order in which he visits the cities so that the **total of the distances travelled** in his tour is as small as possible.

Artwork based on LaValle, Ch. 6

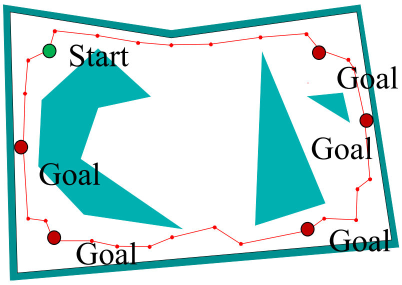
Travelling Salesman Problem

- Given a $n \times n$ distance matrix **C**=*(c_{ij})*

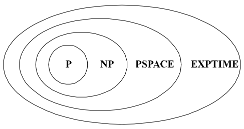
- Minimize:

$$c(\pi) = \sum_{i=1}^{n} c_{i\pi(i)}$$

- Note that this problem is NP-Hard



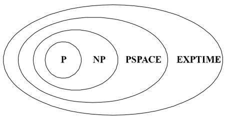
Artwork based on LaVale, Ch. 6



➔ BUT, Special Cases are Well-Solvable!

Travelling Salesman Problem [2]

- This problem is NP-Hard



➔ BUT,
Special Cases are
Well-Solvable!

For the Euclidean case
(where the points are on the 2D Euclidean plane) :

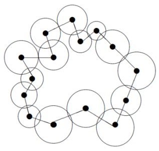
- The shortest TSP tour does not intersect itself, and thus geometry makes the problem somewhat easier.
- If all cities lie on the boundary of a convex polygon, the optimal tour is a cyclic walk along the boundary of the polygon (in clockwise or counterclockwise direction).

***The k*-line TSP**

- The a special case where the cities lie on *k* parallel (or almost parallel) lines in the Euclidean plane.
- EG: Fabrication of printed circuit boards
- Solvable in **O**(n^3) time by Dynamic Programming (Rote's algorithm)

The necklace TSP

- The special Euclidean TSP case where there exist *n* circles around the *n* cities such that every cycle intersects exactly two adjacent circles

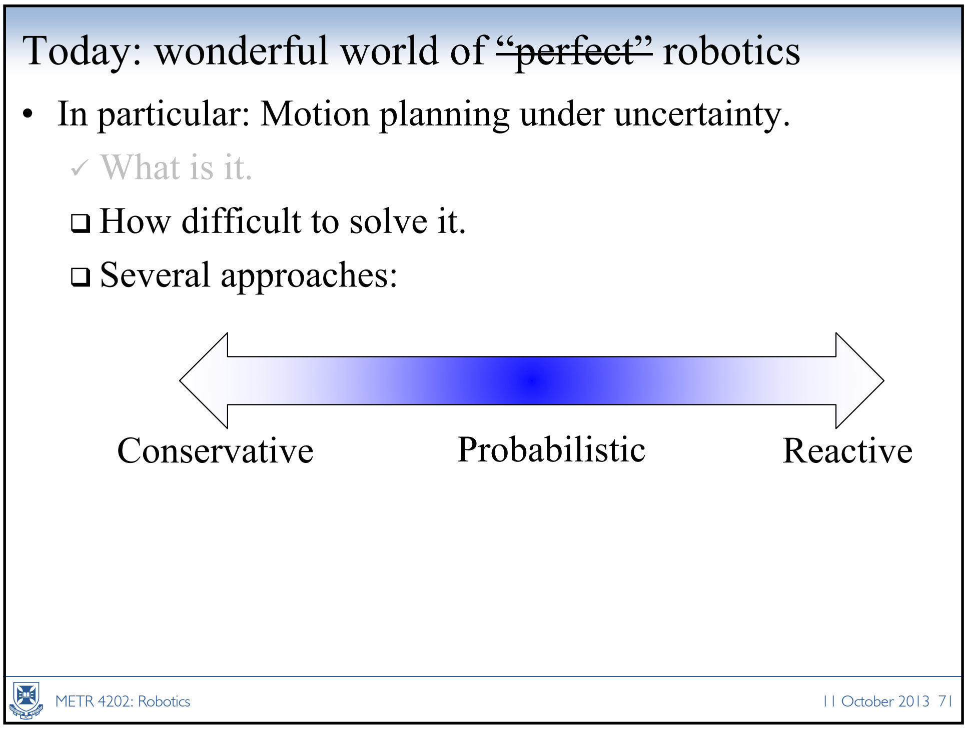


Motion PLANNING & Uncertainty

(Slides C/O Hanna Kurniawati)

Deterministic motion planning vs motion planning under uncertainty

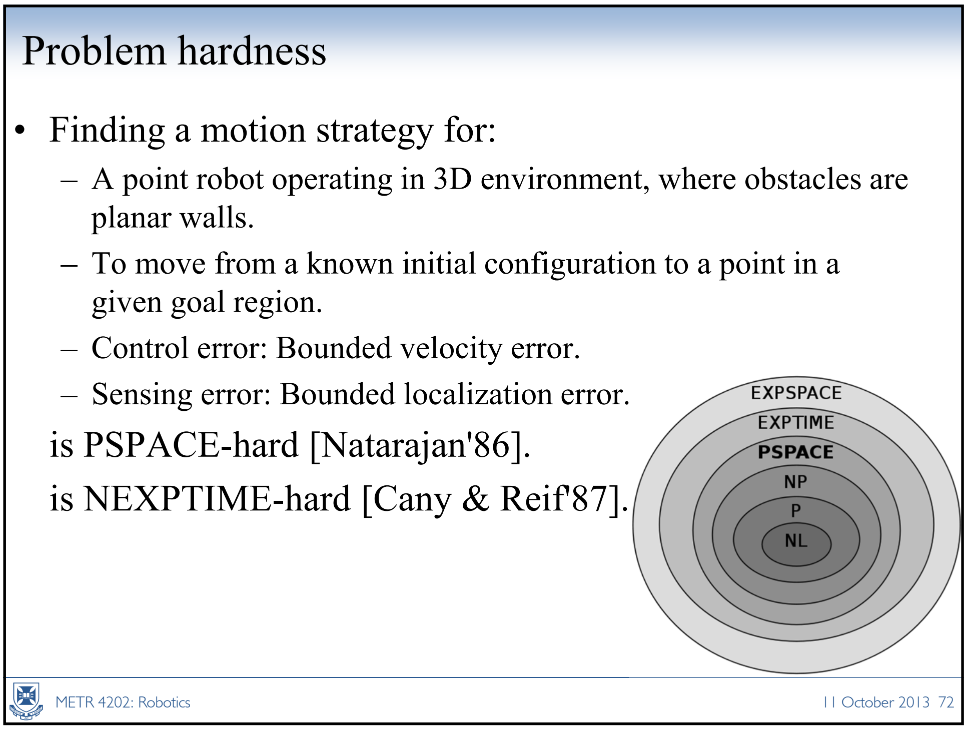
- Deterministic motion planning
  - Find a valid path between two configurations in order to accomplish a task, given:
  - No control error.
  - No sensing.
  - Know the operating environment perfectly.
- Motion planning under uncertainty (today)
  - Find a motion strategy to accomplish a task, where there's a combination of:
  - Control error.
  - Sensing error.
  - Partially / unknown operating environment.



Today: wonderful world of ~~“perfect”~~ robotics

- In particular: Motion planning under uncertainty.
  - ✓ What is it.
  - ❑ How difficult to solve it.
  - ❑ Several approaches:

Conservative Probabilistic Reactive



Problem hardness

- Finding a motion strategy for:
  - A point robot operating in 3D environment, where obstacles are planar walls.
  - To move from a known initial configuration to a point in a given goal region.
  - Control error: Bounded velocity error.
  - Sensing error: Bounded localization error.

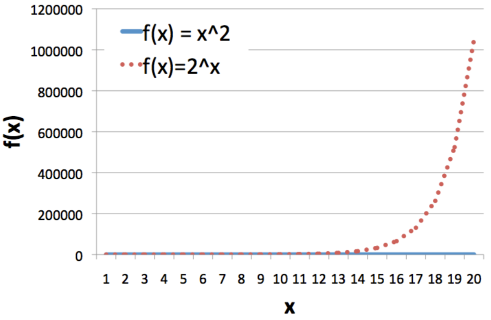
  is PSPACE-hard [Natarajan'86].

  is NEXPTIME-hard [Cany & Reif'87].

EXPSPACE
EXPTIME
PSPACE
NP
P
NL

A little bit on computational complexity 1/3

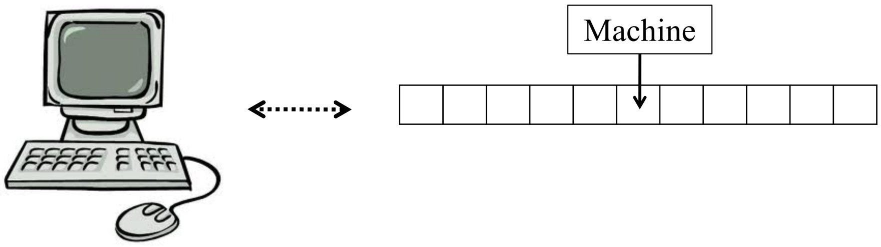
- Algorithms are **not** made to be used only once & are **not** made to be used for only one particular problem.
- How long does it take for the algorithm to find the solution when the input size increases ?
  - In particular, is it polynomial or exponential ?



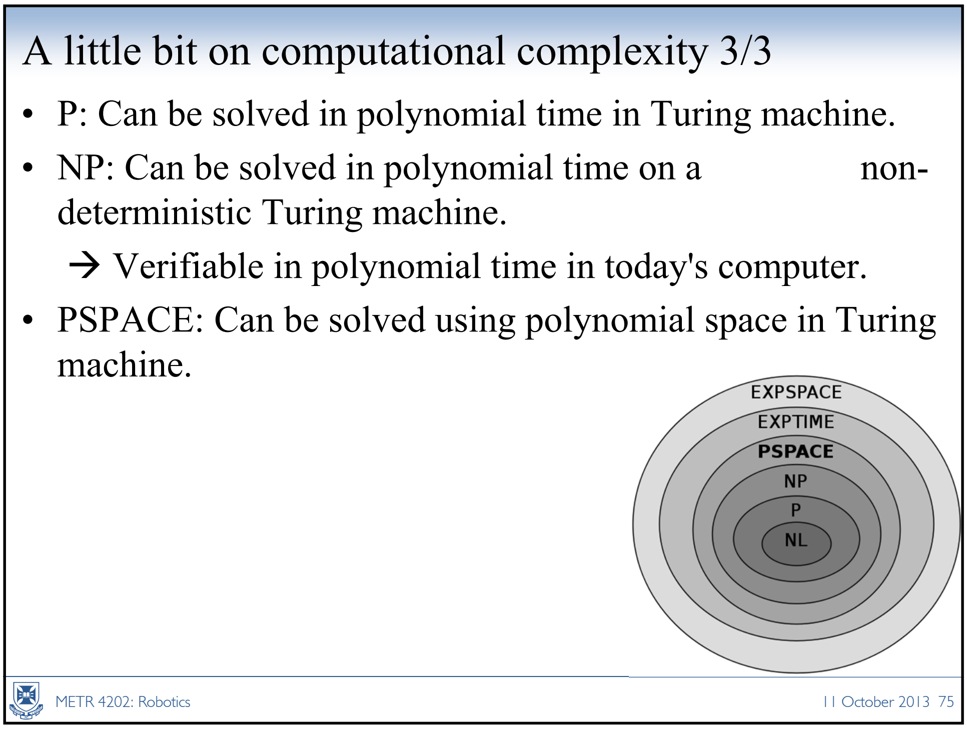
A little bit on computational complexity 2/3

Today’s computer

(Deterministic) Turing machine



Non-Deterministic Turing machine
It can generate multiple possible program executions at once.
Same capability as Turing machine, but can get things done faster.

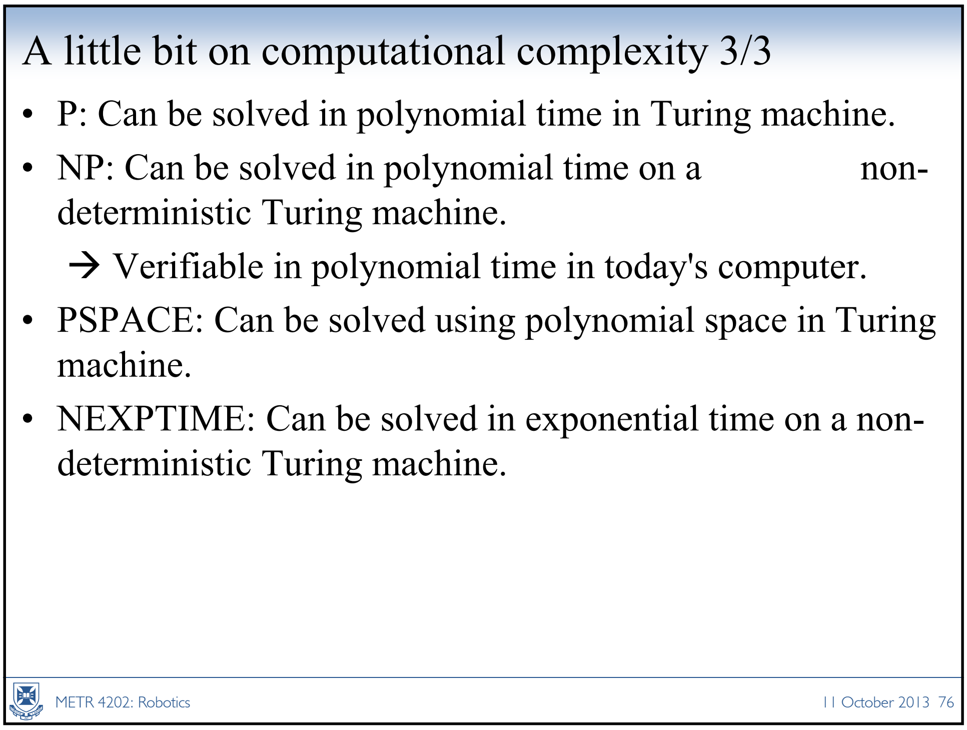


A little bit on computational complexity 3/3

- P: Can be solved in polynomial time in Turing machine.
- NP: Can be solved in polynomial time on a non-deterministic Turing machine.

  → Verifiable in polynomial time in today's computer.
- PSPACE: Can be solved using polynomial space in Turing machine.

EXPSPACE
EXPTIME
PSPACE
NP
P
NL



A little bit on computational complexity 3/3

- P: Can be solved in polynomial time in Turing machine.
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- PSPACE: Can be solved using polynomial space in Turing machine.
- NEXPTIME: Can be solved in exponential time on a non-deterministic Turing machine.

A little bit on computational complexity 3/3

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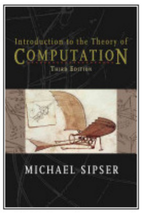
Want to know more ?
Introduction to the Theory of Computation
by Michael Sipser.



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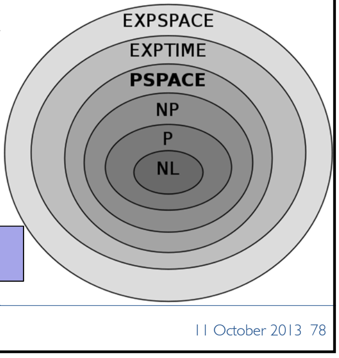
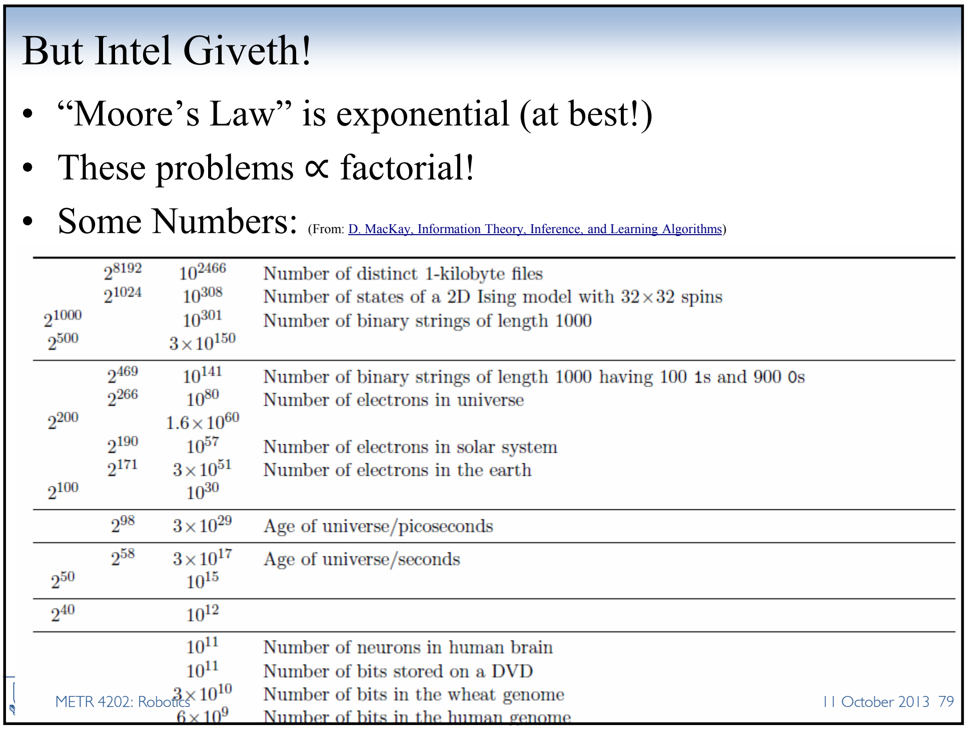
  is PSPACE-hard [Natarajan'86].

  is NEXPTIME-hard [Cany & Reif'87].

  input size: number of planar walls.



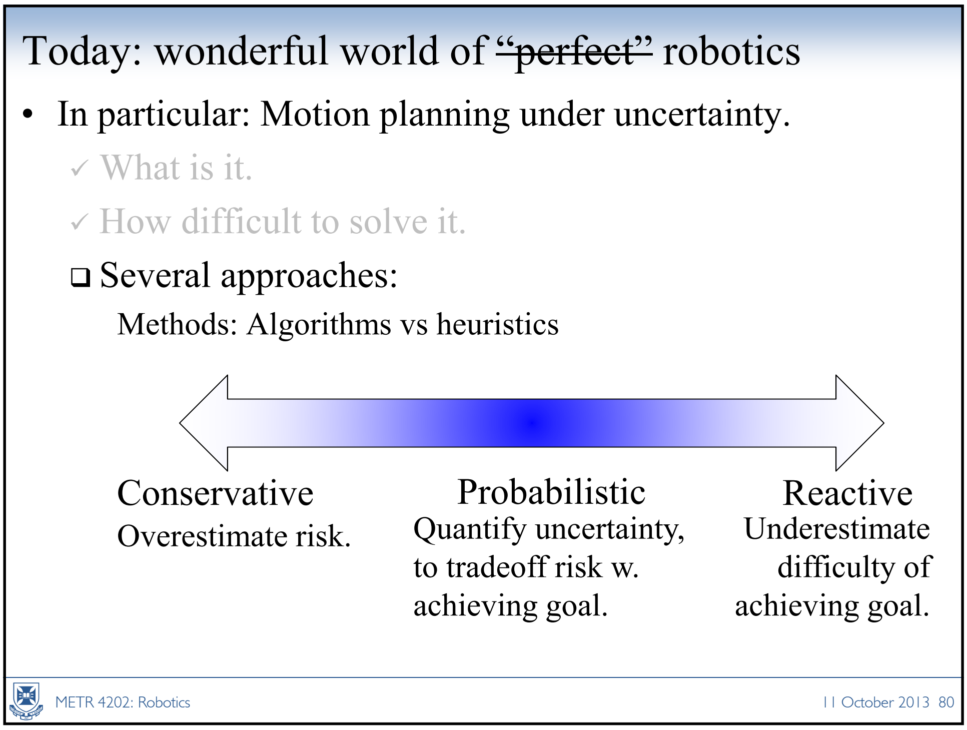
Ok, it's hard… So, what should we do ?



But Intel Giveth!

- "Moore's Law" is exponential (at best!)
- These problems ∝ factorial!
- Some Numbers: (From: D. MacKay, Information Theory, Inference, and Learning Algorithms)

	2^{8192}	10^{2466}	Number of distinct 1-kilobyte files
	2^{1024}	10^{308}	Number of states of a 2D Ising model with 32×32 spins
2^{1000}		10^{301}	Number of binary strings of length 1000
2^{500}		3×10^{150}	
	2^{469}	10^{141}	Number of binary strings of length 1000 having 100 1s and 900 0s
	2^{266}	10^{80}	Number of electrons in universe
2^{200}		1.6×10^{60}	
	2^{190}	10^{57}	Number of electrons in solar system
	2^{171}	3×10^{51}	Number of electrons in the earth
2^{100}		10^{30}	
	2^{98}	3×10^{29}	Age of universe/picoseconds
	2^{58}	3×10^{17}	Age of universe/seconds
2^{50}		10^{15}	
2^{40}		10^{12}	
		10^{11}	Number of neurons in human brain
		10^{11}	Number of bits stored on a DVD
		3×10^{10}	Number of bits in the wheat genome
		6×10^{9}	Number of bits in the human genome



Today: wonderful world of ~~"perfect"~~ robotics

- In particular: Motion planning under uncertainty.
  - ✓ What is it.
  - ✓ How difficult to solve it.
  - ❑ Several approaches:

    Methods: Algorithms vs heuristics

Conservative
Overestimate risk.

Probabilistic
Quantify uncertainty, to tradeoff risk w. achieving goal.

Reactive
Underestimate difficulty of achieving goal.

Probability review 1/4: Probabilistic Modeling

- View:
  - Experiments with random outcome.
  - Quantifiable properties of the outcome.
- Three components:
  - Sample space: Set of all possible outcomes.
  - Events: Subsets of sample space.
  - Probability: Quantify how likely an event occurs.

Probability review 2/4: Probability

- Probability: A function that maps events to real numbers satisfying these axioms:
  1. Non-negativity: $P(E) \quad 0$, where E is an ever
  2. Normalization: $P(S) \quad 1$, where S is the sample space.
  3. Additivity of finite / countably infinite events.

$$P \ \bigcup_{i\ 1}^{/n} E_i \qquad \underset{i\ 1}{\overset{/n}{}} \ P \ E_i \ ,$$

where E $_i$ are disjoint / mutually exclusive, i : natural number.

Probability review 3/4: Random Variables

- Interest is on numerical values associated w. samples, e.g.:
  - Sample 50 students enrolled in METR4202, what's the major of most of the students.
  - Roll a fair dice, get $5 if the outcome is even, & loose $5 if the outcome is odd.
- Random variable X is a function $X : S \rightarrow Num$.
  - Num: countable set (e.g., integer) → discrete random variable.
  - Num: uncountable set (e.g., real) → continuous random variable.

Probability review 4/4: Characterizing Random Variables

- Cumulative distribution function (cdf)

$$F_X(x) = P(X \le x) = P(\{s|X(s) \le x, s \in S\})$$

- Discrete: Probability mass function (pmf)

$$f_X(x) = P(X = x)$$

- Continuous: Probability density function/probability distribution function (pdf)

$$f_X(x) = \frac{dF_X(x)}{dx} \quad ; \quad P(a \le X \le b) = \int_a^b f_X(x)\,dx$$

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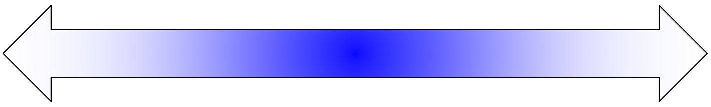

✓Conservative

Probabilistic

POMDP

✓Probability review.

- POMDP.
- POMDP solvers.

✓Reactive

Cool Robotics Share

Universal Gripper

U. Chicago, Cornell, iRobot
May 2010