



This exam has FIVE (5) Problems for a total of 100 Marks:

Problem 1: Rotation Matrices .....	10 %
Problem 2: Kinematics .....	15 %
Problem 3: Dynamics & Motion Planning.....	25 %
Problem 4: Vision.....	25 %
Problem 5: State-Space Control .....	25 %

**Please answer all questions in the Answer Booklet.**

No credit will be awarded for answers on this examination paper.

You may request additional answer booklets if needed. Please note the booklet number out of the total number of booklets used on the first page (e.g., “Booklet 1 of 2”). Please remember to include your name and student number on the first page as well.

Please note the **Extra Credit questions are Difficult!**

Please try not to spend too much time on these questions until you have completed the rest of the exam.

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Material for the exam will come from:

- Tutorial and Tutorial Problems (70%)
- Lectures (20%)
- Laboratories (10%)

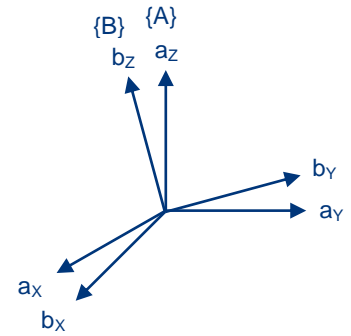
The tutors will provide assistance only if there is real evidence you have attempted the questions

Thank you ☺

1. Rotation Matrices..... [10 marks]

In order to demonstrate your understanding of general rotations in 3D Cartesian space, please provide short answers to the following questions.

- (a) Given unit frames  $\{A\}$  ( $\mathbf{a}_x, \mathbf{a}_y, \mathbf{a}_z$ ) and  $\{B\}$  ( $\mathbf{b}_x, \mathbf{b}_y, \mathbf{b}_z$ ). What are three ways of describing the orientation between them?



- (b) What is the transformation matrix for the case described in (a)?

$${}^A_B\mathbf{T} = \begin{bmatrix} \left[ \begin{array}{ccc} \square & \square & \square \end{array} \right] & \left[ \begin{array}{c} \square \\ \square \\ \square \end{array} \right] \\ \left[ \begin{array}{ccc} \square & \square & \square \end{array} \right] & \left[ \begin{array}{c} \square \\ \square \\ \square \end{array} \right] \\ \left[ \begin{array}{ccc} \square & \square & \square \end{array} \right] & \left[ \begin{array}{c} \square \\ \square \\ \square \end{array} \right] \\ \left[ \begin{array}{ccc} \square & \square & \square \end{array} \right] & \left[ \begin{array}{c} \square \\ \square \\ \square \end{array} \right] \end{bmatrix}$$

- (c) In the following expressions, what is “ $\rho$ ”?

$$\hat{p} = \left[ \rho p_x \quad \rho p_y \quad \rho p_z \quad \rho \right]^T$$

- (d) Very briefly, how is  $SE(n)$  related to  $SO(n)$ ?

2. Kinematics ..... [15 marks]

- a. The figure below shows a 3 link manipulator to be used to retrieve a block from a table.

The arm consists of 3 links with dimensions:

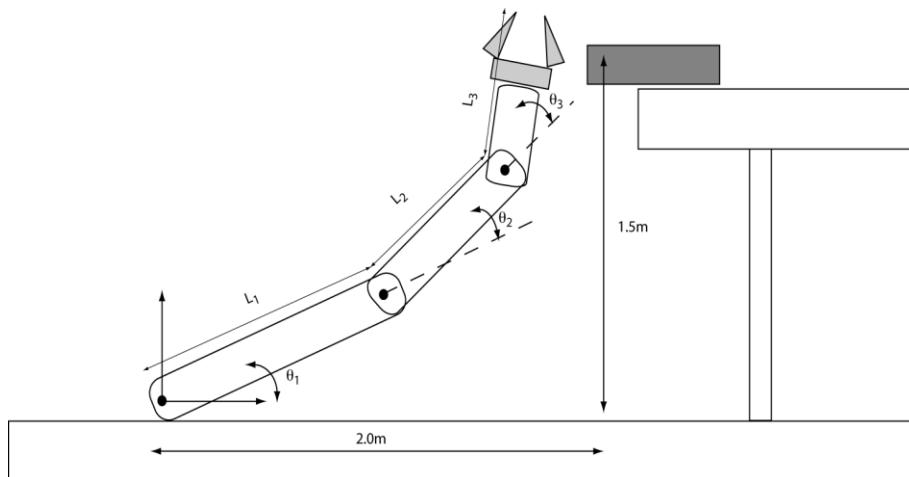
$L_1=1.75$

$L_2=1.25$

$L_3=0.5$

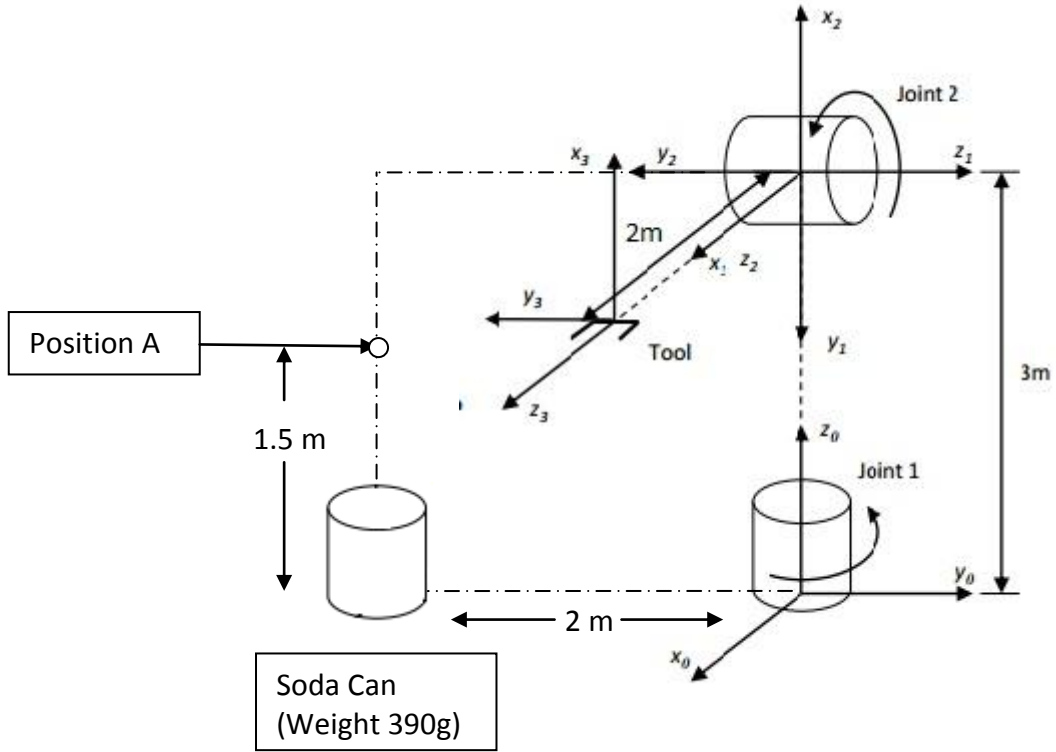
As shown, the block is located at position (2.0, 1.5) relative to the base of the arm. In order to pick up the block, the gripper should be held horizontally with the end effector point around the end of the block.

What joint angles could be used to place the arm in the correct position to retrieve the block?



3. Dynamics and Motion Planning..... [25 marks]

Consider the three link arm below in a **vertical configuration relative to gravity** (i.e., Gravity is pointing into the from the top of the page to the bottom or  $\mathbf{g}=[0\ 0\ -9.8]\ \text{m/s}^2$ )



- a. Derive the equations of motion that describe the system dynamics. Assume that both arm links can be modelled as slender rods with a mass of 0.5kg and 0.3kg and lengths of 3m and 2m respectively.
  
- b. If the arm were to pick up the soda can and hold at position A (assume this is all done for you), then how much Force/Torque will be seen in each joint as a result of the weight of the can?

## 4. Vision..... [25 marks]

a. **Calibration:**

Say we calibrate an undistorted camera with a planar calibration object (such as a chessboard pattern).

What are the terms of the camera calibration matrix?  
(hint: think of the KK matrix in the toolbox)

How does the pixel location ( $x_{\text{image}}, y_{\text{image}}$ ) relate to the parameters above and the metric location of the frame/feature in the scene ( $X, Y, Z$ ).

$$\begin{bmatrix} x_{im} \\ y_{im} \end{bmatrix} = ?$$

b. **More Calibration:**

Can the focal length be recovered by external motion of the camera (e.g., by rotating it around the focal point)? Why or Why not?

c. **Harris Corner and SIFT Features:**

List at least two of the characteristics that Harris features are invariant to.  
(ex. Harris corners are generally unaffected by aperture effects)

List at least two of the characteristics that SIFT features are invariant to and at least one of the ones it is partially invariant to.  
(ex. SIFT Features are partially invariant to illumination changes)

d. **Hough Transform:**

What are the  $\theta$  and  $\rho$  values of a Hough Transform generally correspond to?

## 5. State-Space Control ..... [25 marks]

A quadrotor UAV manoeuvres by changing thrusts on opposite pairs of rotors, and translates by angling the aircraft sideways. Consider a simple planar linear model of quadrotor dynamics:

$$m\ddot{x} = -mg\theta$$

$$I\ddot{\theta} = 2k\delta\omega - 2c\dot{\theta}$$

where  $x$  is horizontal position,  $\theta$  is the pitch angle,  $m$  is the vehicle mass,  $I$  is the vehicle's rotational inertia,  $\tau$  is the combined differential rotor torque,  $k$  is a thrust constant,  $c$  is a rotor inflow damping constant, and  $\delta\omega$  differential rotor velocity (the control input).

- Derive a state-space model of the aircraft's dynamics.
- Draw a nodal block diagram of the state-space system, in control canonical form, clearly marking dynamic states in the model derived in a.
- Design a full-state feedback controller to regulate aircraft lateral position.

**END OF EXAMINATION — Thank you ☺**